

# Business Mathematics

# Contract

# What to expect:

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- Review sections on M or Tu. Quiz most weeks at the beginning of class time.

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- Assignments and Quiz
- One Mid Exam

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- And One Final

The syllabus and all readings and assignments are at:  
<http://lms.iobm.edu.pk>



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Class participation during lecture is strongly encouraged.

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- It is beautiful in both the big picture and in the details and in how they fit together, like an amazing piece of music or poem or car.

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- Algebra.

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These two expressions are equal. What does that mean? We suspect that this equality always holds. You should try to substitute three different sets of values in an equality you do not understand.

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**Example 3.**  $\frac{x^2 - y^2}{x - y} \stackrel{?}{=} x + y.$

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Numbers can represent many different real-world phenomena, but the same rules apply to numbers regardless of what they represent. We will be learning about rules which apply to any kind of function.

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Throughout this course, the domain and range of a function will usually be some collection of numbers and a function is most often denoted by “ $f(x) = \dots$ ”.



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