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Functions can be about measurements of a quantity, such as a cost function which measures for each time the amount it costs to produce a given good.

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Even if you are more interested in functions which measure real-world quantities, part of the power of calculus, and of mathematics in general, is connecting those functions with algebraic functions. For example, both $F = ma$ (Newton) and $E = mc^2$ (Einstein) changed the world, relating fundamental physical quantities through simple functions.

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Example 4. *If $f(x) = \sqrt{x}$ and $g(x) = x^2 - 3x + 2$, what*

is the domain of the function $\frac{f(x)}{g(x)}$*?*

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Example 5. *To measure speed, we take distance travelled and divide it by time. What if we tried to measure the speed of a jet by using a stopwatch over 100 yards?*

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Definition 8. *The slope of the line* $y = mx + b$ *is equal to* m. *It measures the change in* y *if* x *is increased by one. If one is not given the slope explicitly, it can be computed by* $m = \frac{y_2 - y_1}{x_2 - x_1}$ *where* (x_1, y_1) *and* (x_2, y_2) *are*

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Example 9. *Sketch some lines with slope* 1, -1, 2, -3, $\frac{1}{2}$ *and* $\frac{-2}{3}$.

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Example 10. *The points* $(-1,-1)$ *and* $(3,7)$ *are both on the line* $y = 2x + 1$. *We can verify the formula for* m *in this case,and then take two other points on the line and use them to calculate the slope.*

The different descriptions we will use are as follows, listed from simplest to most complicated. We translate each to the standard form, and give an example.

- Standard, slope-intercept form: $y = mx + b$.

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Example 11. *Find the equations - in slope-intercept form - of the following lines:*

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Eq:/line form

- *The line* $3x + 4y = 7$.

$ax + by = c$

$a = 3$

$b = 4$

$c = 7$

eq:

we need the slope intercept form $y = mx + b$

$3x + 4y = 7$

$4y = 7 - 3x$

$y = \frac{7 - 3x}{4} = \frac{7}{4} - \frac{3}{4}x$

$y = -\frac{3}{4}x + \frac{7}{4}$

$m = -\frac{3}{4}$

$b = 7/4$

- *The line* $3x + 4y = 7$.
- *Through the points* $(-1, 4)$ *and* $(2, 3)$.

eg: in two points form is:

$x_1 = -1$, $y_1 = 4$ | $x_2 = 2$, $y_2 = 3$

$$(y - y_1) = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1)$$

$$(y - 4) = \frac{3 - 4}{2 - (-1)}(x - (-1))$$

$$(y - 4) = \frac{-1}{3}(x + 1)$$

slope

$m = -\frac{1}{3}$; $b = ?$

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