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Definition 1. *The slope of the line $y = mx + b$ is equal to m. It measures the change in y if x is increased by one. If one is not given the slope explicitly, it can be computed by $m = \frac{y_2 - y_1}{x_2 - x_1}$ where (x_1, y_1) and (x_2, y_2) are any two points on the line.*

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Example 3. *The points* $(-1, -1)$ *and* $(3, 7)$ *are both on the line* $y = 2x + 1$. *We can verify the formula for* m *in this case,and then take two other points on the line and use them to calculate the slope.*

The different descriptions we will use are as follows, listed from simplest to most complicated. We translate each to the standard form, and give an example.

- Standard, slope-intercept form: $y = mx + b$.

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- Two-point form: "the line between (x_1, y_1) and (x_2, y_2)," namely $(y - y_1) = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$.

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- *The line through the point* $(1, 2)$ *with slope* 3.

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Example 6. *Find the equation of the line perpendicular to the line $2x - 3y = 3$ at the point $(3, 1)$.*

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Question 8. *What about a parabola's defining equation determines whether it opens up or down?*

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We can easily verify the first two statements. The third statement can also be verified through the algebra of "completing the square", but will be very easy to see with a little calculus.

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