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• Find the optimum number of Freshies to produce.

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We develop the function 4^x , to be concrete and to be clear that for exponential functions, the variable appears in the power.

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Example 5. Sketch the graphs of the exponential functions $f(x) = -3 \cdot 2^x$ and $y = (\frac{2}{3})^x$.

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• A personal loan between strangers often has a onetime interest charge. If P dollars is loaned, P + rPdollars is paid back, where r is the interest rate. For example, \$100 loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars.

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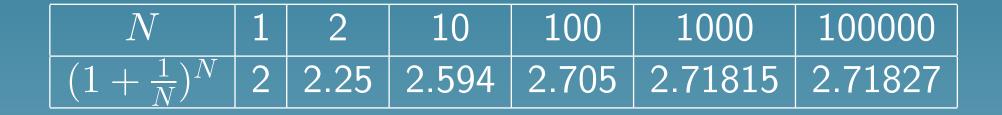
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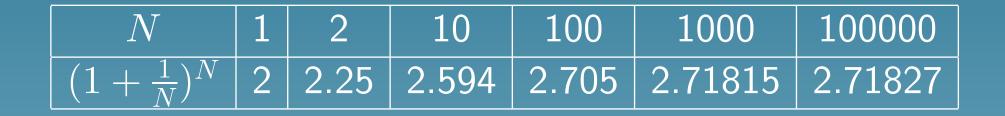
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In the limit, the answer to this question is the magic number e, whose value is approximately 2.71828.

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Example 6. What are you paying out to the credit card company if you have a \$1000 balance at a 19% APR compounded continuously, which you finally pay after two years?

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