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- *Find the optimum number of Freshies to produce.*

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We develop the function 4^x, to be concrete and to be clear that for exponential functions, the variable appears in the power.

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Example 5. *Sketch the graphs of the exponential functions* $f(x) = -3 \cdot 2^x$ *and* $y = (\frac{2}{3})^x$*.*

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In the limit, the answer to this question is the magic number e, whose value is approximately 2.71828.

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Example 6. *What are you paying out to the credit card company if you have a* $\$1000$ *balance at a* 19% *APR compounded continuously, which you finally pay after two years?*

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