Compound interest and the exponential base e

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

- When making a loan to a good friend, you charge no interest.

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

- When making a loan to a good friend, you charge no interest. (If you loan P, you get back P).

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

- When making a loan to a good friend, you charge no interest. (If you loan P, you get back P).

- A personal loan between strangers often has a one-time interest charge.

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

- When making a loan to a good friend, you charge no interest. (If you loan P, you get back P).
- A personal loan between strangers often has a one-time interest charge. If P dollars is loaned, $P + rP$ dollars is paid back, where r is the interest rate.

Compound interest and the exponential base e

Let's review how compounding interest works in different settings.

- When making a loan to a good friend, you charge no interest. (If you loan P, you get back P).
- A personal loan between strangers often has a one-time interest charge. If P dollars is loaned, $P + rP$ dollars is paid back, where r is the interest rate. For

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly. In these cases, one is paying/earning interest on top of interest.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly. In these cases, one is paying/earning interest on top of interest. For example, after two years of 7% interest, $\$100$ becomes $[100 \times (1.07)] \times (1.07) = 114.49$ dollars.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly. In these cases, one is paying/earning interest on top of interest. For example, after two years of 7% interest, $\$100$ becomes $[100 \times (1.07)] \times (1.07) = 114.49$ dollars. (in this case, the 49 cents is the interest on top of interest).

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly. In these cases, one is paying/earning interest on top of interest. For example, after two years of 7% interest, $\$100$ becomes $[100 \times (1.07)] \times (1.07) = 114.49$ dollars. (in this case, the 49 cents is the interest on top of interest). In general, if for a principal of P, the value after n years will be $P(1 + r)^n$.

example, $\$100$ loaned with a one-time 7% rate is repaid as $100 + 0.07 \times 100 = 107$ dollars. It will be helpful to rewrite $P + rP$ as $P(1 + r)$.

- Some loans/investments accrue interest yearly. In these cases, one is paying/earning interest on top of interest. For example, after two years of 7% interest, $\$100$ becomes $[100 \times (1.07)] \times (1.07) = 114.49$ dollars. (in this case, the 49 cents is the interest on top of interest). In general, if for a principal of P, the value after n years will be $P(1 + r)^n$. Note that interest on top of interest becomes much more significant as n gets larger.

- In order to be more sensitive to when transactions are made, it makes sense to compound interest more frequently.

- In order to be more sensitive to when transactions are made, it makes sense to compound interest more frequently. To compound things monthly, we would charge an interest of $\frac{r}{12}$ each month, resulting in a total return of $P(1+\frac{r}{12})^n$, where n is the number of months.

- In order to be more sensitive to when transactions are made, it makes sense to compound interest more frequently. To compound things monthly, we would charge an interest of $\frac{r}{12}$ each month, resulting in a total return of $P(1 + \frac{r}{12})^n$, where n is the number of months. Note that the total for $n = 12$, which is a year, will be greater than if an interest of r percent is changed once.

- In order to be more sensitive to when transactions are made, it makes sense to compound interest more frequently. To compound things monthly, we would charge an interest of $\frac{r}{12}$ each month, resulting in a total return of $P(1+\frac{r}{12})^n$, where n is the number of months. Note that the total for $n = 12$, which is a year, will be greater than if an interest of r percent is changed once. To compound interest every day would result in a total of $P(1+\frac{r}{365})^n$, after n days, for $P(1+\frac{r}{365})^{365}$ after a year.

- We could even compoud every hour, every minute, etc. The general formula is that after a year one has $P(1+\frac{r}{N})^N$, if one compounds N times.

- We could even compoud every hour, every minute, etc. The general formula is that after a year one has $P(1+\frac{r}{N})^N$, if one compounds N times. For a fraction q of a year (such as $\frac{1}{2}$ for six monts), the formula is $P(1+\frac{r}{N})^{qN}$.

- We could even compoud every hour, every minute, etc. The general formula is that after a year one has $P(1+\frac{r}{N})^N$, if one compounds N times. For a fraction q of a year (such as $\frac{1}{2}$ for six monts), the formula is $P(1+\frac{r}{N})^{qN}$. For example, if one compounds 5% yearly interest every day, then after 6 months $\$100$ becomes $100(1+\frac{0.05}{365})^{182} = \$102.53$

- Theoretically, we can compound *continuously* by taking the limit as N goes to ∞ in the formula above.

- Theoretically, we can compound *continuously* by taking the limit as N goes to ∞ in the formula above. For example, suppose we invest $\$1$ with yearly interest of 100%, so that $r = 1$. How much would there be if the interest is compounded continuously?

- Theoretically, we can compound *continuously* by taking the limit as N goes to ∞ in the formula above. For example, suppose we invest $\$1$ with yearly interest of 100%, so that $r = 1$. How much would there be if the interest is compounded continuously?

N	1	2	10	100	1000	100000
$(1 + \frac{1}{N})^N$	2	2.25	2.594	2.705	2.71815	2.71827

- Theoretically, we can compound *continuously* by taking the limit as N goes to ∞ in the formula above. For example, suppose we invest $\$1$ with yearly interest of 100%, so that $r = 1$. How much would there be if the interest is compounded continuously?

N	1	2	10	100	1000	100000
$(1 + \frac{1}{N})^N$	2	2.25	2.594	2.705	2.71815	2.71827

In the limit, the answer to this question is the magic number e, whose value is approximately 2.71828.

Theorem 1. *In general, if P dollars are invested at an annual rate of $r \times 100$ percent, then the balance $B(t)$ after t years is Pe^{rt} dollars.*

Theorem 1. *In general, if P dollars are invested at an annual rate of $r \times 100$ percent, then the balance $B(t)$ after t years is Pe^{rt} dollars.*

The number e is called the natural base for exponentiation, since it occurs raised to a power in problems from almost every quantitative subject.

Theorem 1. *In general, if P dollars are invested at an annual rate of $r \times 100$ percent, then the balance $B(t)$ after t years is Pe^{rt} dollars.*

The number e is called the natural base for exponentiation, since it occurs raised to a power in problems from almost every quantitative subject.

Example 2. *What are you paying out to the credit card company if you have a $\$1000$ balance at a 19% APR compounded continuously, which you finally pay after two years?*

Example 3. *Find the value of* $\$325,000$ *after* 1 *year with* 7% *interest compounded yearly, monthly, daily and continuously.*

Example 3. *Find the value of* $\$325,000$ *after* 1 *year with* 7% *interest compounded yearly, monthly, daily and continuously. (Ans: 347750; 348494.28; 348562.82; 348565.16)*

Example 3. *Find the value of* $\$325,000$ *after* 1 *year with* 7% *interest compounded yearly, monthly, daily and continuously. (Ans: 347750; 348494.28; 348562.82; 348565.16)*

Example 4. *What would you need to deposit in the bank at a 3% interest rate in order to have $40K in the bank ten years from now?*

Example 3. *Find the value of* $\$325,000$ *after* 1 *year with* 7% *interest compounded yearly, monthly, daily and continuously. (Ans: 347750; 348494.28; 348562.82; 348565.16)*

Example 4. *What would you need to deposit in the bank at a 3% interest rate in order to have $40K in the bank ten years from now? (This kind of calculation is called a* present value *calculation).*

Logarithmic functions

Logarithmic functions

Many common functions arise through “undoing” basic functions

Logarithmic functions

Many common functions arise through “undoing” basic functions (more formally, we say they are inverses of basic functions).

Logarithmic functions

Many common functions arise through “undoing” basic functions (more formally, we say they are inverses of basic functions). For example, subtraction was invented as the inverse of addition,

Logarithmic functions

Many common functions arise through "undoing" basic functions (more formally, we say they are inverses of basic functions). For example, subtraction was invented as the inverse of addition, division as the inverse of multiplication,

Logarithmic functions

Many common functions arise through “undoing” basic functions (more formally, we say they are inverses of basic functions). For example, subtraction was invented as the inverse of addition, division as the inverse of multiplication, and the square root as the inverse of the squaring function.

Logarithmic functions

Many common functions arise through "undoing" basic functions (more formally, we say they are inverses of basic functions). For example, subtraction was invented as the inverse of addition, division as the inverse of multiplication, and the square root as the inverse of the squaring function.

Definition 5. *The inverse of the exponential function* a^x *is called the logarithm function (with a base of* a*) denoted* $\log_a(x)$*.*

Logarithmic functions

Many common functions arise through "undoing" basic functions (more formally, we say they are inverses of basic functions). For example, subtraction was invented as the inverse of addition, division as the inverse of multiplication, and the square root as the inverse of the squaring function.

Definition 5. *The inverse of the exponential function* a^x *is called the logarithm function (with a base of* a*) denoted* $\log_a(x)$*. By this definition,* $\log_a a^x = x$ *and* $a^{\log_a x} = x$*.*

The logarithm with a base of e *is called the natural log function and is denoted* $\ln(x)$.

The logarithm with a base of e *is called the natural log function and is denoted* $\ln(x)$.

Example 6. • $\log_{10} 10000 = 6$ *because* $10^6 = 100000$.

The logarithm with a base of e is called the natural log function and is denoted $\ln(x)$.

Example 6. • $\log_{10} 10000 = 6$ *because* $10^6 = 100000$.

• $\log_2 \frac{1}{8} = -3$ *because* $2^{-3} = \frac{1}{8}$.

The logarithm with a base of e is called the natural log function and is denoted $\ln(x)$.

Example 6.

- $\log_{10} 10000 = 6$ *because* $10^6 = 100000$.
- $\log_2 \frac{1}{8} = -3$ *because* $2^{-3} = \frac{1}{8}$.
- $\ln \sqrt{e} = \frac{1}{2}$ *because* $e^{\frac{1}{2}} = \sqrt{e}$.

The logarithm with a base of e is called the natural log function and is denoted $\ln(x)$.

Example 6. • $\log_{10} 10000 = 6$ *because* $10^6 = 100000$.

• $\log_2 \frac{1}{8} = -3$ *because* $2^{-3} = \frac{1}{8}$.

• $\ln \sqrt{e} = \frac{1}{2}$ *because* $e^{\frac{1}{2}} = \sqrt{e}$.

The properties of the logarithm function follow from those of the exponential function.

- $\log_a(xy) = \log_a x + \log_a y$

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a x^y = y \log_a x$

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a x^y = y \log_a x$ follows from $(a^x)^y = a^{xy}$.

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a x^y = y \log_a x$ follows from $(a^x)^y = a^{xy}$.

- $\log_b x = \log_b a \cdot \log_a x$

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a x^y = y \log_a x$ follows from $(a^x)^y = a^{xy}$.

- $\log_b x = \log_b a \cdot \log_a x$ follows from the previous property.

In other words, logarithms turn multiplication to addition and turn exponentiation to multiplication.

- $\log_a(xy) = \log_a x + \log_a y$ follows from $a^{x+y} = a^x a^y$.

- $\log_a x^y = y \log_a x$ follows from $(a^x)^y = a^{xy}$.

- $\log_b x = \log_b a \cdot \log_a x$ follows from the previous property.

In other words, logarithms turn multiplication to addition and turn exponentiation to multiplication. Also note that the last property says that logarithms with different bases are related by multiplication by a constant.

Logarithms are handy in problems involving exponentials.

Logarithms are handy in problems involving exponentials.

Example 7. *A radioactive material decays at a rate of* 0.5% *per year. How long would it take for half of the material to decay?*

Logarithms are handy in problems involving exponentials.

Example 7. *A radioactive material decays at a rate of* 0.5% *per year. How long would it take for half of the material to decay?*

Because of these properties, logarithms played an important role in nerd history -

Logarithms are handy in problems involving exponentials.

Example 7. *A radioactive material decays at a rate of* 0.5% *per year. How long would it take for half of the material to decay?*

Because of these properties, logarithms played an important role in nerd history - before calculators, nerds always carried slide rules because slide rules calculate logarithms, which may be used to do long calculations by hand.

Logarithms are handy in problems involving exponentials.

Example 7. *A radioactive material decays at a rate of* 0.5% *per year. How long would it take for half of the material to decay?*

Because of these properties, logarithms played an important role in nerd history - before calculators, nerds always carried slide rules because slide rules calculate logarithms, which may be used to do long calculations by hand.

Example 8. *Use logarithms to calculate:* 365×843

Example 8. *Use logarithms to calculate:* 365×843