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Ans of Q. 1

Given that

$\sum_{i,j=1}^{n} a_{ij}$, where $a_{ij} = 0$ for $i \nless j$

For expanding the above expression, we can write $\sum_{i,j=1}^{n} a_{ij}$ in the form of matrix

i.e

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & a_{24} & \cdots & a_{2n} \\ 0 & 0 & 0 & a_{34} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & & 0 \end{bmatrix}$$

Now change the matrix A in determinant form

P.T.O

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$$A = \begin{bmatrix} 0 & a_{12} & a_{13} & \cdots & a_{1n} \\ 0 & 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$|A| = 0 \begin{vmatrix} 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0_{nn} \end{vmatrix} - a_{12} \begin{vmatrix} 0 & a_{23} & \cdots & a_{2n} \\ 0 & 0 & \cdots & a_{3n} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0_{n\times n} \end{vmatrix} + \cdots - a_{1n} \begin{vmatrix} 0 & 0 & \cdots & a_{2n-1} \\ 0 & 0 & \cdots & a_{3n-1} \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0_{n-1} \end{vmatrix}$$

$$-a_{1n} \begin{vmatrix} 0 & 0 & \cdots & a_{2n-1} \\ 0 & 0 & \cdots & a_{3n-1} \\ 0 & 0 & \cdots & a_{n-1,n} \end{vmatrix}$$

1. Using result that determinant is equal to product of eigenvalues and product of eigenvalues is equal to trace of matrix. [Trace of matrix is sum of diagonal engtries], therefore determinant of given matrix is zero.
2. Since $\det(A) = \det(A^T)$, and while considering A^T, one can notice that all cofactors are basically multiplied by zero, hence zero determinant.

$$|A| = 0 - 0 + \cdots + 0$$

$$|A| = 0$$

Remark:→ If a matrix having last Row is equal to zero (0) then its determinant is also be zero (0)

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Q.4 :→ Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ \end{bmatrix}$

5

3

Ans of Q. 2

Given that

$\sum_{i,j=1}^{n} a_{ij}$, where $a_{ij}=0$ for $i \geq j$

Firstly, for expanding, we can write the above expression in the form of matrix

i.e.

$$A = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ a_{21} & 0 & 0 & \cdots & 0 \\ a_{31} & a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & a_{n3} & \cdots & 0 \end{bmatrix}_{n \times n}$$

Same arguments as given in Q No. 1

Now, from finding determinant we can find the value of a

$$|A| = 0 \begin{vmatrix} 0 & 0 & \cdots & 0 \\ a_{32} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n} & \cdots & a_{nn} \end{vmatrix} - 0 \begin{vmatrix} a_{21} & 0 & \cdots & 0 \\ a_{31} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} + \cdots - 0 \begin{vmatrix} a_{21} & a & \cdots & 0 \\ a_{31} & a_{32} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{n(n-1)} \end{vmatrix}$$

$|A| = 0$

Hence the Sumation $\sum_{i,j=1}^{n} a_{ij} = 0$ for $a_{ij}=0 \; \forall \, i \geq j$

Q.4 :

Ans of Q. 3

$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 12 & 10 & 10 \end{bmatrix}$ find its determinant by elementary row operation?

Sol:

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Sol:→ change the above matrix in the form of determinant

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 12 & 10 & 10 \end{vmatrix}$$

$R_2 - 9R_1 \rightarrow R_2$

$R_3 - 12R_1 \rightarrow R_2$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -14 & -26 \\ 0 & -14 & -26 \end{vmatrix}$$

$R_3 + R_2 \rightarrow R_3$

$$|A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & -14 & -26 \\ 0 & 0 & 0 \end{vmatrix}$$

Determinant of any matrix A is equation of its row equivalent matrix ??

Performing one row operation (operation of adding two rows) perserves determinant. Other two two operations does not perserve determinant. Multiplying a row scales determinant and exchanging two rows scales determinant with -1.

$|A| = 0$

From original matrix the determinant is also 0

Q.4 :→ Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 12 & 10 & 10 \end{bmatrix}$ (5)

Sol:→ Its obvious from solution of Q.3 that this matrix has 2 rank

Here, I am introducing 2 methods of finding rank which are given below.

1) By elementary row operation

i.e.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 12 & 10 & 10 \end{bmatrix}$$

$R_2 - 9R_1 \rightarrow R_2$

$R_3 - 12R_1 \rightarrow R_3$

$$A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -14 & -26 \\ 0 & -14 & -26 \end{bmatrix}$$

$R_3 + R_2$

$$A_3 = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -14 & -26 \\ 0 & 0 & 0 \end{bmatrix}$$

Hence the rank is $\boxed{2}$

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6

2) By determinant method.

Convert A matrix in the determinant

$$A = \begin{vmatrix} 1 & 2 & 3 \\ 9 & 4 & 1 \\ 12 & 10 & 10 \end{vmatrix}$$

$|A| = 1(+30) - 2(78) + 3(42)$

$|A| = +30 - 156 + 126$

$|A| = 15\cancel{6} - 15\cancel{6}$

$|A| = 0$

Hence from $|A|$ it is obvious that the rank of a matrix A is not equal to 3

Now take any minor of that matrix if it is equal to non-zero, it means that it has 2 rank

$$M_{11} = \begin{vmatrix} 4 & 1 \\ 10 & 10 \end{vmatrix} \Rightarrow 30$$

$M_{11} \neq 0$

It means that the matrix A has rank equal to $\boxed{2}$