بسم الله الرحمن الرحيم

Faraz Mukhtiar Rajput.

Roll No: F.18 BS(MS)32

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Group: Math - Mrates S.

Dept: Mathematics & Statistics.

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Q: No: 1 : find the values of x. for which matrix $A = \begin{bmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{bmatrix}$ is investable.

Sol

A matrix is investable iff determinant of Matrix not equal to zero.
for instance: A is any square matrix and inverse of A exist iff

$$\det(A) \neq 0$$

So In Question first we will find the det of matrix A.

$$\det(A) = \begin{vmatrix} 1 & 1 & x \\ 1 & x & x \\ x & x & x \end{vmatrix}$$

$$= 1(x^2 - x^2) - 1(x - x^2) + x(x - x^2)$$

$$= -x + x^2 + x^2 - x^3$$

$$\det(A) = -x + 2x^2 - x^3 \neq 0$$

Now we will find the values of x.

$$-x + 2x^2 - x^3 = 0 \Rightarrow -x^3 + 2x^2 - x = 0$$

By using Synthetic division method we will find the values of x.

$$\begin{array}{r|rrr}
1 & -1 & 2 & -1 \\
 & & -1 & 1 \\
\hline
 & -1 & 1 & \boxed{0}
\end{array}$$

$$-x^2 + x = 0$$

$$-x(+x-1) = 0$$

$$-x = 0$$

$$x = 1$$

values of $x = \{0, 1\}$

Now we put the values of x in matrix by using theorem: Condition that yields a zero determinant:

Conditions:

1. An entire row or column consist of zero
2. Two row or column equal
3. One row or column is multiple of another row or column.

we can put the values of x in Matrix "A" except these condition. then matrix will be invertable.

So Matrix of A is

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

What value of x did you use? I think, you used different values at same time??

Use single value of x, for which matrix is invertible.

$$|A| = \begin{vmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{vmatrix}$$

$$= 1(0-1) - 1(1-0) + 0(1-0)$$

$$= -1-1$$

$$\boxed{|A| = -2}$$ Answer.

Q: No: 2: Find inverse of matrix A by putting values of x for which A is in-vertable

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

solution: Being by adjoining the identity matrix to A to form the matrix

$$[A : I] = \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$= \left[\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \quad R_3 - R_1 \Rightarrow R_3$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & -1 & 1 & 0 \end{array}\right] \quad R_2 + R_3 \Rightarrow R_2$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \\ 0 & -1 & 1 & -1 & 1 & 0 \end{array}\right] \quad R_2 \leftrightarrow R_3$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 2 & -1 & 1 & 1 \end{array}\right] \quad \frac{1}{2} R_3 \Rightarrow R_3$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 1 & -1 & 1 & 0 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right] \quad -R_2 \Rightarrow R_2$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right] \quad R_2 + R_3 \Rightarrow R_2$$

$$= \left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right] \quad R_1 - R_2 \Rightarrow R_2$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 & 1/2 & -1/2 & 1/2 \\ 0 & 0 & 1 & -1/2 & 1/2 & 1/2 \end{array}\right]$$

The matrin A is investable and it's inverse is

$$A^{-1} = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 1/2 & -1/2 & 1/2 \\ -1/2 & 1/2 & 1/2 \end{bmatrix}$$

Answer

Q: No: 3 Find the determinant of matrices

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix} \quad \text{and} \quad U = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

Sol

1. $$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

By using Determinant of a Triangular theorem.

$$|L| = 1(1)(1) = 1$$

$$|L| = 1$$

which is the product of the entire on the main diagonal.

Solve

2. $$U = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

find $|U|$ by using determinant of a triangular theorem

$$|U| = 1(1)(1) = 1$$

$$|U| = 1$$

which is the product of the entire on the main diagonal.

Q:No: 4 Find the inverse of matrices in Q:No: 3 using elementary row operation.

Sol

$$L = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ b & c & 1 \end{bmatrix}$$

Solution:

Begin by adjoining the identity matrix to A to form the matrix.

$$[L:I] = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ a & 1 & 0 & 0 & 1 & 0 \\ b & c & 1 & 0 & 0 & 1 \end{array}\right] \begin{array}{l} R_2 - aR_1 \Rightarrow R_2 \\ R_3 - bR_1 \Rightarrow R_3 \end{array}$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -a & 1 & 0 \\ 0 & c & 1 & -b & 0 & 1 \end{array}\right] R_3 - cR_2 \Rightarrow R_3$$

$$[L:I] = \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -a & 1 & 0 \\ 0 & 0 & 1 & -b+ca & -c & 1 \end{array}\right]$$

$$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -a & 1 & 0 \\ -b+ca & -c & 1 \end{bmatrix}$$

Answer.

2.
$$U = \begin{bmatrix} 1 & a & b \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

Sol.

Beign by adjoining the identity matrix to A to form the matrix.

$$[U : I] = \left[\begin{array}{ccc:ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & c & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad R_2 - cR_3 \Rightarrow R_2$$

$$= \left[\begin{array}{ccc:ccc} 1 & a & b & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right] \quad \begin{array}{l} R_1 - aR_2 \Rightarrow R_1 \\ R_1 - bR_3 \Rightarrow R_1 \end{array}$$

$$= \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 0 & -a & -(ac+b) \\ 0 & 1 & 0 & 0 & 1 & c \\ 0 & 0 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$U^{-1} = \begin{bmatrix} 0 & -a & -(ac+b) \\ 0 & 1 & c \\ 0 & 0 & 1 \end{bmatrix}$$

Answer.