**Comment: Overall good. Lot of typos and spelling mistakes.**

Q No. 1 **Show that set of all polynomials of degree at-most 2 with scalars from** $R,$$P\_{2}\left[R\left(x\right)\right]=\left\{a\_{2}x^{2}+a\_{1}x+a\_{0}\right|a\_{i}\in R ∀i=0,1,2\} $**is vector space over field ℝ**

Solution:

First we have to check that either given polynomial is vector space or not.so we have to *check* all (TEN) properties

1. (u+v) is close under addition . where u+v ∈V

 p(x) = a2x2+a1x1+a0

q(x) = b2x2+b1x1+b0

Now we have to add these above equations as;

 p(x) +q(x) = a2x2+a1x1+a0 +b2x2+b1x1+b0

rearrange the same coefficients

 = (a2x2+b2x2) + (a1x1+b1x1) +(a0+b0)

 take common variable

 = x2(a2+b2) + x1(a1+b1) + (a0+b0) where a,b ∈ ℝ

 hence this is closed under addition.

 2. (u+v) = (v+u) Commutative property

 = (a2x2+a1x1+a0 )+(b2x2+b1x1+b0) where u,v ∈ V

 =(b2x2+b1x1+b0) + (a2x2+a1x1+a0 ) = v+u.

 hence u+v=v+u

 3. u+(v+w) = (u+v)w associative property where u,v, and w ∈ V

 = a2x2+a1x1+a0 (+b2x2+b1x1+b0 + c2x2+c1x1+c0 )

 = (a2x2+a1x1+a0 + b2x2+b1x1+b0) +( c2x2+c1x1+c0 )

 hence u+(v+w) = (u+v)+w

 4. u+0 = u additive identity where u ∈ V

 = a2x2+a1x1+a0 +0 = a2x2+a1x1+a0

 hence u+0 = u

 5. u+(-u) = 0 where u ∈ V

 = a2x2+a1x1+a0  + (-a2x2-a1x1-a0 ) = 0

 hence u+(-u) = 0

 6. cu closed under scalar multiplication

 where c,a ∈ ℝ

 c(a2x2+a1x1+a0 ) = c a2x2+ca1x1+ca0

 hence cu is closed under multiplication

 7. c(u+v) = cu + cv distributive property

 where c ∈ ℝ

 c(a2x2+a1x1+a0 +b2x2+b1x1+b0) multiply c inside the brackets

 c a2x2+ca1x1+ca0 +cb2x2+cb1x1+cb0

 c(a2x2+a1x1+a0) + c(b2x2+b1x1+b0)

 cu + cv

 hence c(u+v) = cu + cv

 8. (c+d)u = cu + du distributive property c,d ∈ ℝ

 (c+d) + (a2x2+a1x1+a0 ) = (c a2x2+ca1x1+ca0 ) + (da2x2+da1x1+da0)

 take c and d as common

 c(a2x2+a1x1+a0) + d(a2x2+a1x1+a0)

 cu + du

 hence (c+d)u = cu + du

 9. c(du) = (cd)u associative property c,d ∈ ℝ

 c[d(a2x2+a1x1+a0)] = c( da2x2+da1x1+da0)

 (cda2x2+cda1x1+cda0) take c and d as common then (cd)a2x2+a1x1+a0

hence c(du) = (cd)u

10. 1.u = u

 1.(a2x2+a1x1+a0) = (a2x2+a1x1+a0)

 hence

QNO.02 Define ***spain*** of a set in vector space. Show that span of any set in vector space is subspace over same field of vector space.

ANS The set of all linear combinations of elements of S is called the LINEAR SPAN. And it is denoted by ⟨S⟩

Let V be vector space and S⊂V where S ={v1,v2,v3...vn}

Liner span is also defiend as all linear combination of vi, i=1,2,3...neach vector vi in S is linear combination of v1,v2,v3...vnV

Vi = 0v1+0v2+0v3...+0vi-1+1vi+0vi+1+...0vn

Thus ⟨S⟩ contains each vectors vi belonging to S i.e S⊂⟨S⟩

Q NO.03 Consider set of four polynomials S = {p1(x) = 1+x+x2, p2(x) = x+2x2, p3(x) = -1 and p4(x) = x2}.

Show that span (S) is subspace of P2[ℝ(x)].

ANS V = ℝ4 = ℝ2× ℝ2 = {(x,y), (x,y)}

S ={(1+x+x2),( x+2x2), (-1), (x2)}

Span (s) = av1 + bv2 + cv3 + dv4

= a(1+x+x2) + b(x+2x2) + c(-1) + d (x2)

a +ax+ax2 + bx+2bx2-c+dx2

s1,s2,s3,s4 ∈⟨S⟩

S1 = a1(1+x+x2) + b1( x+2x2) + c1(-1) + d1 (x2)

S2 = a2(1+x+x2) + b2( x+2x2) + c2(-1) + d2 (x2)

S3= a3(1+x+x2) + b3( x+2x2) + c3(-1) + d3 (x2)

S4= a4(1+x+x2) + b4( x+2x2) + c4(-1) + d4 (x2)

S1 = a1+a1x+a1x2 + b1x+2b1x2 –c1+d1x2

S2 = a2+a2x+a2x2 + b2x+2b2x2 –c2 +d2x2

S3 = a3+a3x+a3x2 + b3x+2b3x2 –c3 +d3x2

S4 = a4+a4x+a4x2 + b4x+2b4x2 –c4 +d4x2

⇒ a1+a1x+a1x2 + b1x+2b1x2 –c1+d1x2 + a2+a2x+a2x2 + b2x+2b2x2 –c2 +d2x2 + a3+a3x+a3x2 + b3x+2b3x2 –c3 +d3x2 + a4+a4x+a4x2 + b4x+2b4x2 –c4 +d4x2

(a1+ a2+ a3+ a4) + (a1x + a2x + a3x + a4x) + (a1x2 + a2x2 + a3x2 + a4x2) +( b1x + b2x + b3x + b4x) +( 2b1x2 +2b2x2 +2b3x2 +2b4x2 ) + (–c1 –c2 –c3–c4) + (d1x2 + d2x2 + d3x2 + d4x2)

⇒ (a1+ a2+ a3+ a4) + x(a1+ a2+ a3+ a4) + x2(a1+ a2+ a3+ a4)

⇒ x(b1+ b2+ b3 + b4) +2 x2(b1+ b2+ b3 + b4)

⇒(–c1 –c2 –c3–c4)

⇒ x2 (d1 + d2+ d3 + d4)

Take (a1+ a2+ a3+ a4), (b1+ b2+ b3 + b4) (–c1 –c2 –c3–c4), (d1 + d2+ d3 + d4) as common then;

[(a1+ a2+ a3+ a4) (1+x+x2)] +[(b1+ b2+ b3 + b4) ( x+2x2)] –[ (c1 +c2 +c3+c4)( –1)] +[(d1 + d2+ d3 + d4)( x2)]

Let suppose (a1+ a2+ a3+ a4) = a

 (b1+ b2+ b3+ b4)=b

 (c1 +c2 +c3+c4)=c

 (d1 + d2+ d3 + d4)=d

a(1+x+x2) + b( x+2x2) +(–c ) + d(x2).

Q NO.04 Construct Basis of S in Q N0.03 from given four polynomials (vectors).

ANS Let p2 be the vector space of all polynomials with real coefficients of degree at most 2.

Let S= {p1I(x),p2(x),p3(x),p4(x)}

Where

 p1(x) = 1+x+x2, p2(x) = x+2x2, p3(x) = -1 and p4(x) = x2

find a basis of p2 among the vectors of S.

The vector space p2 has a basis B ={1,x,x2} and the dimension of p2  is 3.

The co\_ordinate vectors with respect to this basis are

[p1(x)]B = $\left(\begin{matrix}1\\1\\1\end{matrix}\right)$ [p2(x)]B = $\left(\begin{matrix}0\\1\\2\end{matrix}\right)$

[p3(x)]B = $\left(\begin{matrix}0\\0\\-1\end{matrix}\right)$ [p4(x)]B = $\left(\begin{matrix}1\\0\\0\end{matrix}\right)$

Consider the vector space span (T)

T ={ [p1(x)]B, [p2(x)]B, [p3(x)]B, [p4(x)]}

We determine a basis vector of span (T) among vectors in T,

Consider the matrix A whose column vectors are the co-ordinates vectors

A = $\left(\begin{matrix}1&0& 0 1 \\1&1& 0 0 \\1&2&-1 0 \end{matrix}\right)$ R2-R1,R3-R1 $\left(\begin{matrix}1&0& 0 1 \\0&1& 0 -1 \\1&1&-1 0 \end{matrix}\right)$R3-R2

$$\left(\begin{matrix}1&0& 0 1 \\1&1& 0 -1 \\1&2& 1 1 \end{matrix}\right)$$

{ [p1(x)]B, [p2(x)]B, [p3(x)]B, [p4(x)]} is a BASIS of the vector space span (T).



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