Team:

Irrationals

Roll no:

F18MS-38
F18MS-03
F18MS-68
F18MS-18

Q.NO 1: Show that $T: P_2[\mathbb{R}(x)] \to \mathbb{R}^3$ defined by $T(ax^2+bx+c) = (a,b,c)$ is linear Transformation.

Proof: let $U = a_1x^2+b_1x+c_1$

$V = a_2x^2+b_2x+c_2$

Now

$$\alpha U+V = \alpha(a_1x^2+b_1x+c_1)+(a_2x^2+b_2x+c_2)$$

$$T(\alpha U+V) = T((\alpha a_1+a_2)x^2+(\alpha b_1+b_2)x+(c_1+c_2))$$

According to given condition **Ccondition or definition of T?**

$$T(ax^2+bx+c) = (a,b,c)$$

$$= (\alpha a_1+a_2, \alpha b_1+b_2, \alpha c_1+c_2)$$

$$T(\alpha U+V) = T(\alpha a_1, \alpha b_1, \alpha c_1)+(a_2,b_2,c_2)$$

$$= \alpha T(a_1,b_1,c_1)+(a_2,b_2,c_2)$$

hence both the conditions are satisfied

QNO 2: Write the matrix corresponding to linear Transformation in QNO 1 with respect standard basis of $P_2[\mathbb{R}(x)]$ and $\mathbb{R}^3$.

As from QNO 1 we know that

$T: P_2[\mathbb{R}(x)] \to \mathbb{R}^3$ is defined as

$$T(ax^2+bx+c) = (a,b,c)$$

let suppose B and E are standard basis respectively of $P_2[\mathbb{R}(x)]$.

This is standard basis ?

$$B = \{\underbrace{(0x^2+0x+c)}_{e_1}, \underbrace{(0x^2+bx+c)}_{e_2}, \underbrace{(ax^2+bx+c)}_{e_3}\}$$

$$B = \{e_1, e_2, e_3\}$$

$$\mathbb{R}^3 = E = \{(1,0,0),(0,1,0),(0,0,1)\}$$

$$T(e_1) = 0(1,0,0) + 0(0,1,0) + c(0,0,1) = (0,0,c)$$

$$T(e_2) = 0(1,0,0) + b(0,1,0) + c(0,0,1) = (0,b,c)$$

$$T(e_3) = a(1,0,0) + b(0,1,0) + c(0,0,1) = (a,b,c)$$

so the matrix is

$$\begin{bmatrix} 0 & 0 & a \\ 0 & b & b \\ c & c & c \end{bmatrix}$$

Not correct matrix!

How will you multiply this matrix to a polynomial (vector) instead of applying T ?

Q:3 Show that set $\beta = \{-1, 1+x^2, 1+x+x^2\}$ is basis of $P_2[\mathbb{R}(x)]$.

Proof: To prove whether the set is basis or not. we will prove whether given set is linearly dependent or Independent ?
if the given set is linearly independent then we are done with it.

Let $a, b, c \in \mathbb{R}$ then

$$= a(-1) + b(1+x^2) + c(1+x+x^2) = 0$$

$$= -a + b + bx^2 + c + cx + cx^2 = 0$$

$$= (b+c)x^2 + (c)x + (-a+b+c) = 0$$

$b+c=0$ $\Rightarrow$ $b+0=0$

$\boxed{c=0}$ so $\boxed{b=0}$

$-a+b+c=0$ $\quad$ $-a-0-0=0$

$\boxed{a=0}$

Do these three polynomials span the complete basis? Remember spanning is second condition.

from above equations it is clear that
$a=b=c=0$
so the vectors given in set are linearly independent so given set B is basis of $P_2[\mathbb{R}(x)]$

QNO 4 Write the matrix of linear Transformation in QNO 1 with respect to basis of $P_2[\mathbb{R}(x)]$ in QNO 3 and standard basis of $\mathbb{R}^3$.

solution:- As from QNO 1 know that

$T: P_2[\mathbb{R}(x)] \to \mathbb{R}^3$ defined by

$$T'(ax^2+bx+c) = (a,b,c)$$

Now the given basis of $P_2[\mathbb{R}(x)]$ in QNO 3 defined by set β and standard basis of $\mathbb{R}^3$ by P we get

$$B = \{\underbrace{(0x^2+0x-1)}_{e_1}, \underbrace{(x^2+0x+1)}_{e_2}, \underbrace{(x^3+x+1)}_{e_3}\}$$

$$\beta = \{e_1, e_2, e_3\}$$

$$P = \{(1,0,0), (0,1,0), (0,0,1)\}$$

Now

$$T(e_1) = 0(1,0,0) + 0(0,0,1) + 1(0,0,1) = (0,0,-1)$$

$$T(e_2) = 1(1,0,0) + 0(0,1,0) + 1(0,0,1) = (1,0,1)$$

$$T(e_3) = 1(1,0,0) + 1(0,1,0) + 1(0,0,1) = (1,1,1)$$

so the matrix will be

$$\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$

Same comment as in Q No. 2