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Team Mathsketeers

Roll Numbers:-

F18MS-61

F.18MS-62

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Q.NO:1 Consider vector space of all real valued function defined on interval $I=[-1,1]$. check if set of function (vectors) $\{\sqrt{x}, x\}$ is linear independent.

Solution:-

Let $v_1, v_2 \in V$, $a_1, a_2 \in F$ (scalars).

$S=\{v_1, v_2\}$

$v_1 = (\sqrt{x_1}, x_1)$ and $v_2 = (\sqrt{x_2}, x_2)$ These two are vectors , why x_1 and x_2

Now we will check the two vectors are linear independent, So that any vector in V (vector space) is the linear Combination of other two vectors.

$a_1 V_1 + a_2 V_2 = u$ $\qquad a_1, a_2 \in F$

Do it again!!

$a_1(\sqrt{x_1}, x_1) + a_2(\sqrt{x_2}, x_2) = (0,0)$

$a\sqrt{x_1}, a_1 x_1 + a_2\sqrt{x_2} + a_2 x_2 = (0,0)$

$a\sqrt{x_1} + a_2\sqrt{x_2}, a_1 x_1 + a_2 x_2 = (0,0)$

By equating, we have

$$a_1\sqrt{x_1} + a_2\sqrt{x_2} = 0 \longrightarrow (i)$$

$$a_1x_1 + a_2x_2 = 0 \longrightarrow (ii)$$

$\therefore$ a_1 and a_2 will be equal to zero and then put equations (i) or (ii) equations will be satisfied.

$\Rightarrow$ $a_1 = a_2 = 0$

By the def of linear independent all scalars are equal to zero. It means these vectors ~~are~~ v_1 and v_2 are linear independent. ANS:

Q#02: By adding another function x^2 in set in Q No: 1 check linear independence or dependence.

Solution:

$$S = \{v_1, v_2\}$$

let $v_1, v_2 \in V$ and a_1, a_2 are scalars over F

You are NOT adding set, but in coordinate??

$v_1 = (\sqrt{x_1}, x_1, x_1^2)$ and $v_2 = (\sqrt{x_2}, x_2, x_2^2)$ By Ques: No

Misconcepted!!!

Now these ~~ui~~ vectors will be check linear independence or dependence So that any vector "u" in V is the uniquely written as the linear combinat of other vectors v_1 and v_2

Q#03: Show that no set of two vectors can be basi[s] vector space $\mathbb{R}^3$

Solution:-

$$S = \left\{ \begin{array}{l} v_1 = (1,1,2) \\ v_2 = (0,1,7) \end{array} \right\}$$

A quick solution is to note that any basis of $\mathbb{R}^3$ must consist Of three vectors, Thus can not be a basis as S contain two only vectors. and v is any vector is equal to (a,b,c) is in span(S) if and only if v is a linear Combination of vectors in S.

$$v = a_1v_1 + a_2v_2 \qquad a_1, a_2 \in F, \quad v_1, v_2 \in V$$

$$(a,b,c) = a_1(1,1,2) + a_2(0,1,7)$$

$$(a,b,c) = 1a_1, 1a_1, 2a_1 + 0a_2 + 1a_2, 7a_2$$

$$(a,b,c) = 1a_1 + 0a_2, 1a_1 + 1a_2, 2a_1 + 7a_2$$

By equating, we have.

03

So,

$$a_1 V_1 + a_2 V_2 = u \qquad a_1, a_2 \in F$$

$$a_1(\sqrt{x_1}, x_1, x_1^2) + a_2(\sqrt{x_2}, x_2, x_2^2) = (0,0,0)$$

$$a\sqrt{x_1}, ax_1, ax_1^2 + a_2\sqrt{x_2}, a_2x_2, a_2x_2^2 = (0,0,0)$$

$$a_1\sqrt{x_1} + a_2\sqrt{x_2} = 0 \longrightarrow (i)$$

$$a_1x_1 + a_2x_2 = 0 \longrightarrow (ii)$$

$$a_1x_1^2 + a_2x_2^2 = 0 \longrightarrow (iii)$$

a_1 and a_2 will be equal to zero and then put the equations $\underline{i}$ and $\underline{ii}$ and then equations will be satisfied.

$\Rightarrow \quad a_1 = a = 0$

By def say all scalars are equal to zero

It means these vectors ~~are~~ V_1 and V_2 are linearly Independence.

Two vectors are linearly independent then why can not be basis?

$\Rightarrow \quad V_1, V_2$ are Linearly Independence.

Q4: Find bases Of real vector space $\mathbb{R}$.

Solution:-

Given set

$S = \{v_1\}$

$\therefore V_1 = 1$

$\Rightarrow S = \{1\}$ Of the vector in the vector space $\mathbb{R}$ find a basis for Span S.

The set $S = \{v_1\}$ of vectors in $\mathbb{R}$, Only one element $v_1 = 1$ It is also linearly independent.

By the linear Combination i-e.

$C_1 V_1 = 0$

$\Rightarrow \boxed{C_1 = 0}$

In this case, the set S forms a basis for span S

i-e
$$a_1 = a_1 V_1$$
$$a_2 = a_2 V_1$$
$$a_3 = a_3 V_1$$
$$\vdots \qquad \vdots$$
$$a_n = a_n V_1$$

$1a_1 + 0a_2 = a$
$1a_1 + 1a_2 = b$
$2a_1 + 7a_2 = c.$

$$\begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 2 & 7 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix} \text{ is consistent.}$$

let us consider the augmented matrix and reduce it by elementry row operations.

$$\Rightarrow \left[\begin{array}{cc|c} 1 & 0 & a \\ 1 & 1 & b \\ 2 & 7 & c \end{array}\right] \quad \begin{array}{l} R_2 - R_1 \to R_2 \\ R_3 - 2R_1 \to R_3 \end{array} \left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b-a \\ 0 & 7 & c-2a \end{array}\right]$$

$$\Rightarrow R_3 - 7R_2 \to R_3 \left[\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b-a \\ 0 & 0 & 5a-7b+c \end{array}\right].$$

Note that we obtained the (3,3) entry by

$$c - 2a - 7(b-a) = 5a - 7b + c.$$

It follows that the system is consistent if and if

$$5a - 7b + c = 0$$

Thus, for example, the vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ is not in span(S).

Ok.

as $5 \cdot 1 - 7 \cdot 0 + 0 \neq 0$

Hence span(S) is not $\mathbb{R}^3$, and we conclude that S is not a basis.

Hence shown.