بسم الله الرحمن الرحيم

In the Name of Allah,
the Beneficent, the Merciful.

SUBJECT: Linear Algebra

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Q:1: Differentiate B/w geometric & Algebric multiplicity of eigen values:.

Geometric multiplicity: - dimension of Eigen space for the eigen value λ. $\dim(E_A(\lambda))$ and How many Basis vector

geometric multiplicity Formula: $\underset{\substack{3\times3\\2\times2}}{\overset{\text{matrix}}{n}} - \overset{\text{Rank}}{\rho}(A)$.

Ex: $A: \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (1)

Find Gauss Elimination.

$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \text{Rank} = 2.$

$g.m = n - \rho(A)$

$3 - 2 = 1 \rightarrow g.m.$

Algebraic multiplicity of λ: number of times of λ: or: roots of characteristic polynomial.

Def: multiplicity of λ is a root of characteristic polynomial of A.

Ex: $\begin{bmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow (1-\lambda)(3-\lambda)^{2} \rightarrow A.M$

Eigenvalue of Algebric multiplicity λ: $(3-\lambda)^2 = \lambda = 3$ is 2 $\rightarrow$ A.M

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Fact:
Geometric multiplicity $\lambda \le$ Algebric multiplicity λ

Notice:- If You an eign value that has Algebric multiplicity 1 automatically its Geometric multiplicity is 1.

Rank: non Zero row of reduce row echlon Form:.

nonzero row. Number of linearly independent of

Geometric multiplicity: The Geometric multiplicity of an eign value is the number of linearly Independent eign vector associated with it

$$G.m = n - \delta(A).$$

wher $\delta(A)$ = Rank of that λ specific

$$\therefore \delta(A)\ [A - \lambda I].$$

Q:2 clearly one of the eigen value of matrix

$$A=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$ is 0. without computing all eigen ~~space~~ value, Find Algebraic multiplicity of zero eign value:

$$[A-\lambda I]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}-\lambda\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[A-\lambda I]=\begin{bmatrix} 0-\lambda & 0 & 0 & 0 \\ 1 & 1-\lambda & 1 & 1 \\ 0 & 0 & 0-\lambda & 0 \\ 1 & 1 & 1 & 1-\lambda \end{bmatrix} \Rightarrow \text{Characteristic Matrix}$$

Without computing??

$$\Rightarrow \boxed{\lambda^4-2\lambda^3} \Rightarrow \text{characteristic polynomial.}$$

Read question carefully

$$\Rightarrow \lambda\times\lambda\times\lambda\times(\lambda-2)=\lambda\times\lambda\times\lambda\times(\lambda-2)$$

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(i) $\lambda_1=0$.

(ii) $\lambda_2=2$

Note:

If algebric multiplicity of zero eigen value:

Geometric multiplicity λ: dimension of eigen space correspon[ding] to the eigen value 0. In your case the whole space is an eigen ~~value~~ space corresponding to t[he] eigen value 0. so the geometric multiplicity of the eign value 0 is 2.

geometric multiplicity of an eigen value is th number of linearly independent eigen vector associated in the case the eign associated wi the eigen value in case, the eigen vectors corresponding to the zero eign values.

$AV=0$

Q:3

For Example:- $E_A(\lambda)$ eigen space of A for λ.

$$N(A-2I)$$

$$\cancel{A-2I} \Rightarrow A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & +1 \end{bmatrix}$$

(4)

$$P(A) = (1-\lambda)(2-\lambda)^{2} \rightarrow A.M$$

$$Ex: (A-2I) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow Rank = 2.$$

$$\dim(N(A-2I)) = 1$$

The eigen value is 2 has $g.m = 1$

$$g.m = \underset{3\times 3}{n} - \overset{\rightarrow Rank}{\ell(A)}.$$

$$3-2 = 1$$

Ans:

Q:3:- Define eigen space · if possible:-

Eigen space:-

Def: The set $E\lambda$ of all such eigen vector is a Sub space of V. Called the Eigen space of λ. (if $\dim E\lambda = 1$ then $E\lambda$ is Called an eigen line and λ is Called Scaling Factor.

Def: Let λ be an eigen value of linear operator $T: V \to V$ and Let $E\lambda$ Consists of all the eigen vector Belonging to λ is Called eigen of eigen space A For λ.

For Example: $E_A(\lambda)$ eigen space

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$$A = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$N(A-2I)$$

$$P(A) = \begin{bmatrix} 2-\lambda & 1 & 0 \\ 0 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{bmatrix} \Rightarrow \text{C·Matrix}$$

$$P(A) = (1-\lambda)(2-\lambda)^{2} \xrightarrow{2 \to A.M} \text{Characteristic Polynom}$$

$$Ex: (A-2I) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \Rightarrow \text{Rank} = 2$$

$$\dim(N(A-2I)) = 1$$

Q:4 Find Basis of eign ~~value~~ space

$E_{\lambda=2}$ ~~of Matrix~~ Corresponding to eigen value

$\lambda=2$ of matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ -2 & 4 & 0 \end{bmatrix}$

geometric multiplicity of $\lambda=2$ For

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ -2 & 4 & 0 \end{bmatrix}$$

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$$AV = \lambda V$$

$$[A-\lambda I]V = 0$$

$$[A-\lambda I] = A = \begin{bmatrix} 1 & 2 & 1 \\ -1 & 4 & 1 \\ -2 & 4 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[A-\lambda I] = \begin{bmatrix} 1-\lambda & 2 & 1 \\ -1 & 4-\lambda & 1 \\ -2 & 4 & 0-\lambda \end{bmatrix}$$ => Characteristic matrix.

$$[A-2I] = \begin{bmatrix} 1-(-2) & 2 & 1 \\ -1 & 4-(-2) & 1 \\ -2 & 4 & 0-(-2) \end{bmatrix}$$

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 6 & 1 \\ -2 & 4 & 2 \end{vmatrix}$$

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Find Reduce Row echlon Form:

$$\begin{vmatrix} 3 & 2 & 1 \\ -1 & 6 & 1 \\ -2 & 4 & 2 \end{vmatrix}$$

multiply the $R_1\left(\frac{1}{3}\right)$

$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ -1 & 6 & 1 \\ -2 & 4 & 2 \end{vmatrix}$$

add 1 times to 1st Row to the 2nd row.

$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{20}{3} & \frac{4}{3} \\ -2 & 4 & 2 \end{vmatrix}$$

add 2 times the 1 Row to the 3rd Row.

$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{20}{3} & \frac{4}{3} \\ 0 & \frac{16}{3} & \frac{4}{3} \end{vmatrix}$$

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multiply the 2nd Row by $\frac{3}{20}$

$$\Rightarrow \begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{5} \\ 0 & \frac{16}{3} & \frac{4}{3} \end{vmatrix}$$

add $-\frac{16}{3}$ times the 2nd row to the 3rd row.

$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & \frac{1}{5} \\ 0 & 0 & \frac{4}{15} \end{vmatrix}$$

multiply the 3rd row $\frac{15}{4}$

$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & \frac{1}{3} & \frac{1}{5} \\ 0 & 0 & 1 \end{vmatrix}$$

add $-\frac{1}{5}$ times the 3rd to the 2nd row.

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$$\begin{vmatrix} 1 & \frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

add $-\frac{1}{3}$ times the 3rd row to the 1st row.

$$\begin{vmatrix} 1 & \frac{2}{3} & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

add $-\frac{2}{3}$ times the 2nd row to the 1st row

$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ \end{array}$$
$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

x_1
x_2
x_3

The solution set is $\begin{array}{c} x_1 \\ x_2 \\ x_3 \end{array}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

Geometric multiplicity: dimension of Eigen space $\dim(E_A(\lambda))$

How many Basis vector one.

$\lambda_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

so Basis vector one or one linear independent.

Geometric multiplicity Formula:

which is matrix 2×2 or 3×3 ← $n - \rho(A)$ → Rank

3×3

$3 - 2 = 1$ → This is geometric multiplicity

so linear independent and geometric are same which one.

Note: If you an eign value that has Algebraic multiplicity 1 automatically its geometric multiplicity is 1

O.K

- *Specially Thanks to Sir Abdul Hanan Sheikh*