# On convergence of shifted Laplace preconditioner combined with multigrid deflation

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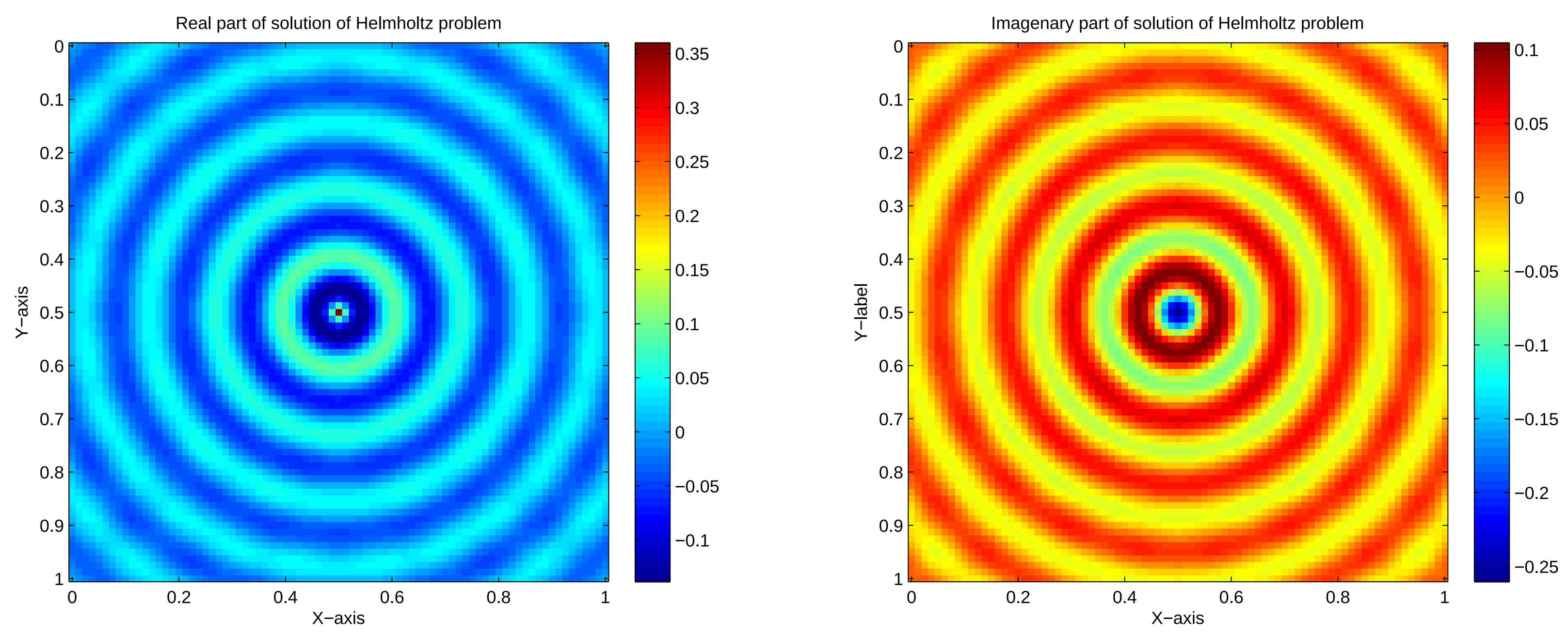


Figure 1: Real *(left)* part and imagenary *(right)* part of solution of the Helmholtz equation solved by GMRES preconditioned with shifted Laplace preconditioner $M(1, 0.1)$.

## Helmholtz Model Problem

The Helmholtz equation with Sommerfeld B.cs. is

$$-\Delta\mathbf{u}(x,y) - k^2\mathbf{u}(x,y)\mathbf{u}(x,y) = \mathbf{g}(x,y)$$
$$(\frac{\delta u}{\delta n} - \iota k u) = 0$$

where $\frac{\delta u}{\delta n}$, the normal derivative of $u$ , $k = \frac{2\pi}{\lambda} = \frac{\omega}{c(x)}$ , the wavenumber and $g(x,y)$ , the point source function.

Discretizaton leads to 5 diagonal, symmetric, complex valued and indefinite linear system.

## Solver

**Two-level** preconditioned **Krylov subspace solvers** i.e. **GMRES**. Shifted Laplace preconditioner performs better than available preconditioners for Helmholtz, and comes up near-zero eigenvalues for large wavenumber probelm.Second level preconditioner:

First level preconditioner : Shifted Laplace Preconditioner

$$M_h := -\Delta - (\beta_1 + \iota\beta_2)k^2 I_h \qquad (1)$$

Second level preconditioner : Multigrid deflation

$$P_{h,H} = I_h - I_H^h(A_H)^{-1}I_h^H A_h \text{ with } A_H = I_h^H A_h I_H^h . \qquad (2)$$

### A Good Characteristic

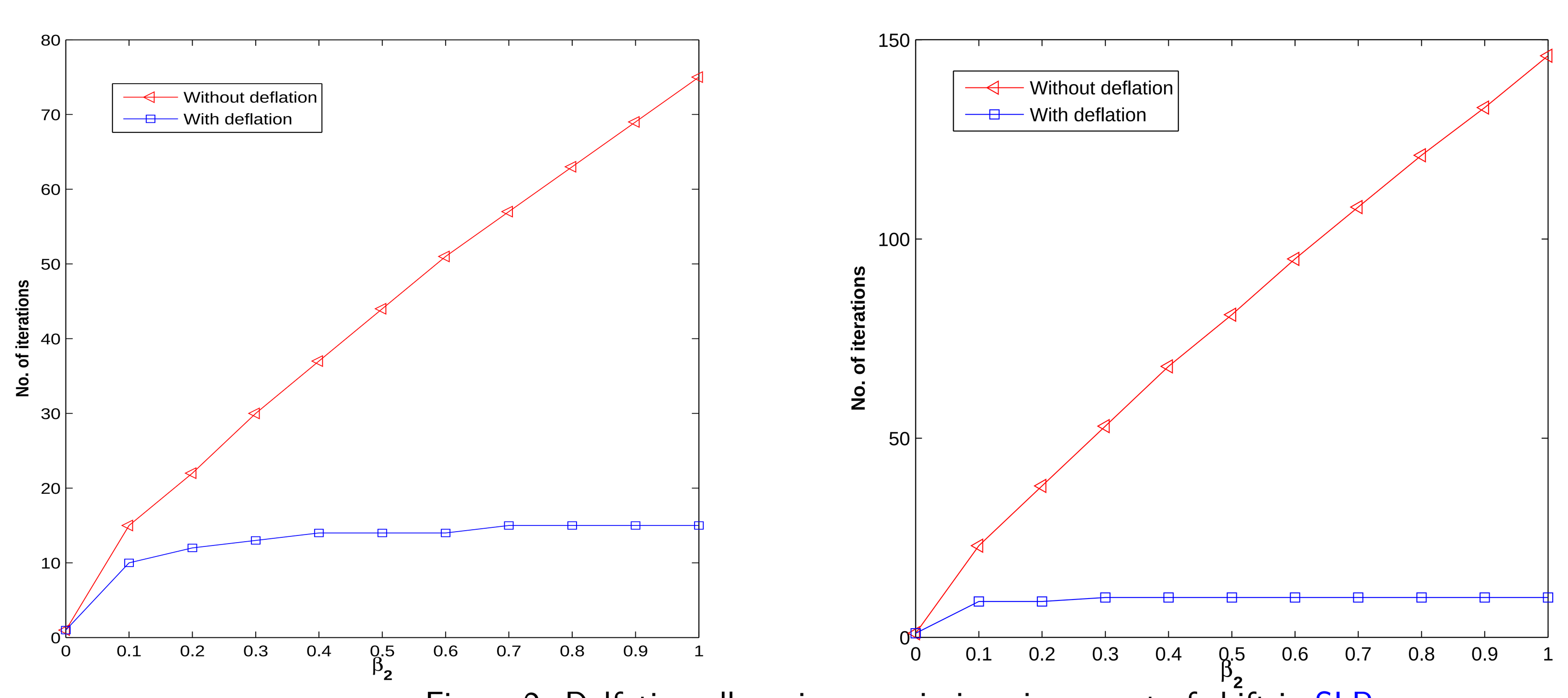


Figure 2: Delfation allows increase in imaginary part of shift in SLP

## Multilevel Results

| | k = 10 | k = 20 | k = 40 | k = 80 | k = 160 | 320 |
|---|---|---|---|---|---|---|
| TL | 6 | 7 | 11 | 15 | 25 | 50+ |
| MLMGV(4, 2, 1) | 9 | 11 | 16 | 27 | 100+ | - |
| MLMGV(4, 2, 1)* | 9 | 11 | 15 | 24 | 50 | - |
| MLMGV(6, 2, 1) | 9 | 10 | 14 | 21 | 47 | - |
| MLMGV(6, 2, 1)* | 9 | 10 | 14 | 20 | 37 | - |
| MLMGV(8, 2, 1) | 9 | 10 | 13 | 20 | 38 | - |
| MLMGV(8, 2, 1)* | 9 | 10 | 13 | 19 | 29 | - |
| MLMGV(10, 2, 1) | 9 | 10 | 14 | 19 | 32 | - |

$*$ with damping $\alpha = 0.001$

## LFA: 2D Model problem

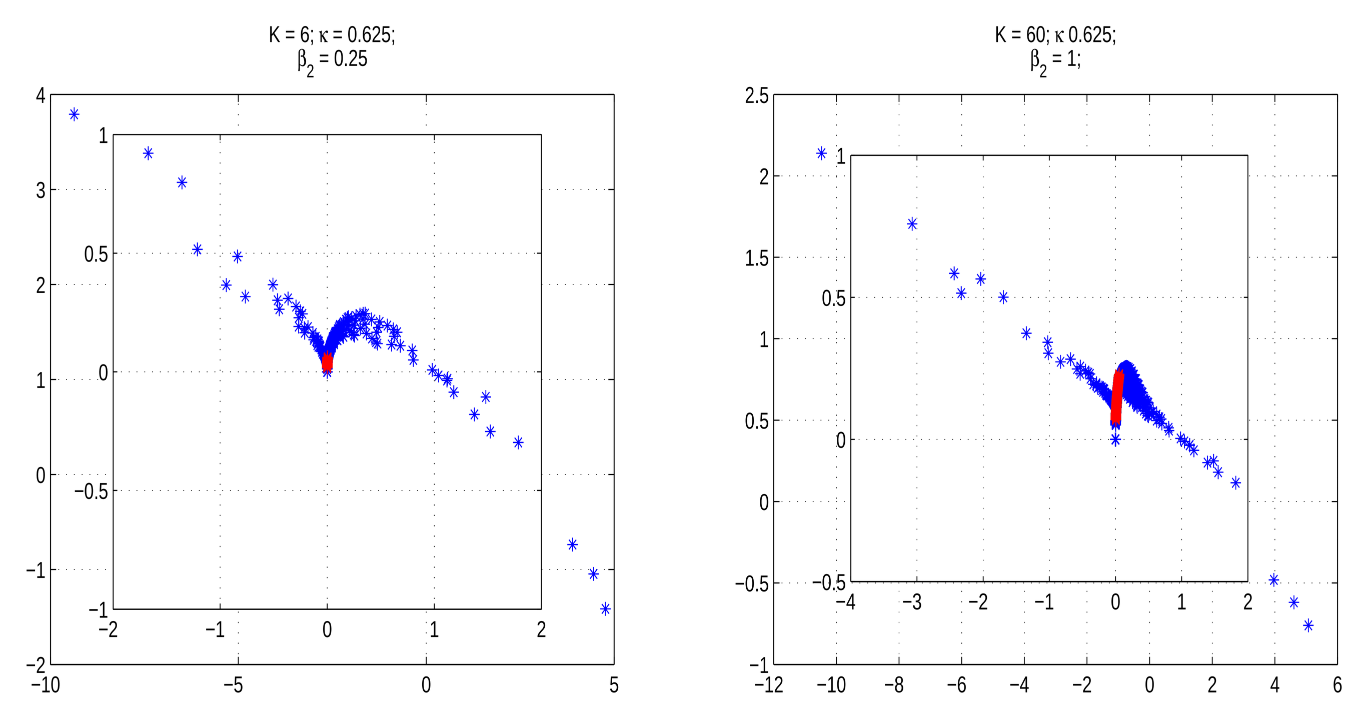

Figure 3: Spectrum of the two grid operator for different values of shift $\beta_2$.

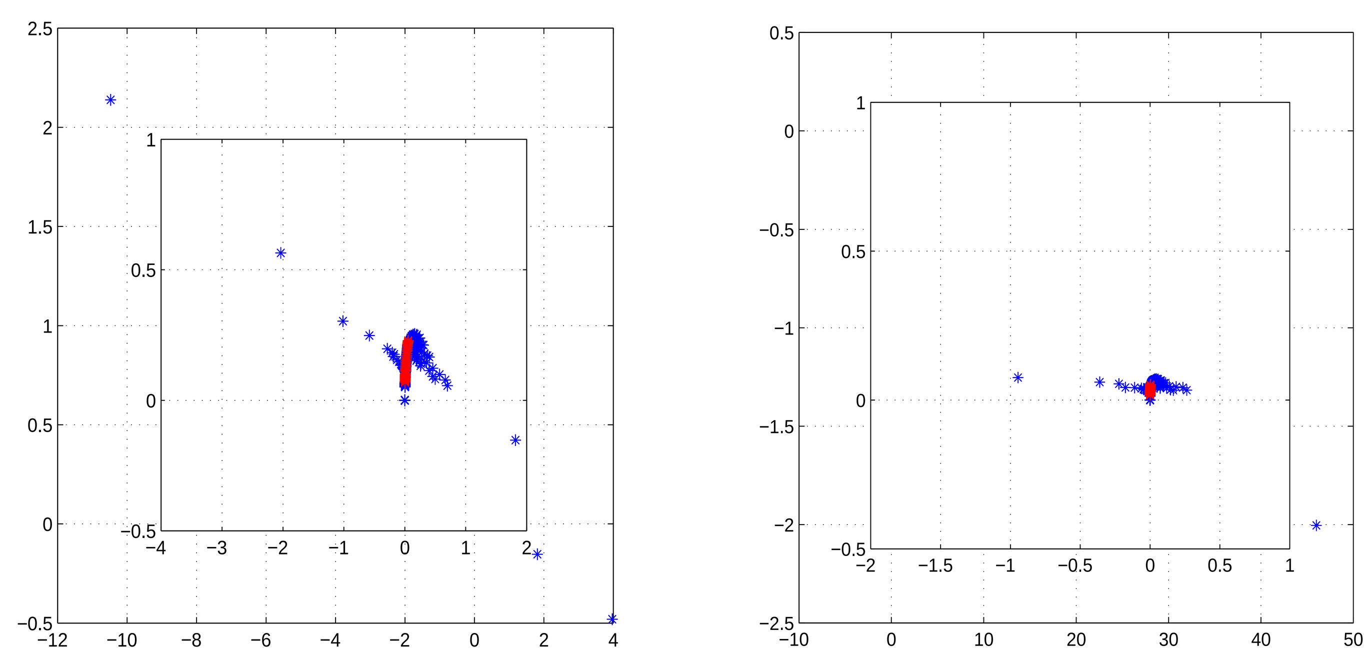

Figure 4: Spectrum of the two grid operator for 10 and 20 gp/wl.

## Conclusive remarks

- Very slightly dependent.
- More wavenumber is resolved over grid, the more efficient algorithm is.
- Coarse grid solve requires more iteration.
- Increase in imaginary part of shift is privileged by deflation.

## References

- Y.A. Erlangga and R. Nabben, ETNA 2008.
- DIAM Tech. Report. 11-01 TU Delft, Netherlands