On convergence of shifted Laplace preconditioner combined with multigrid deflation

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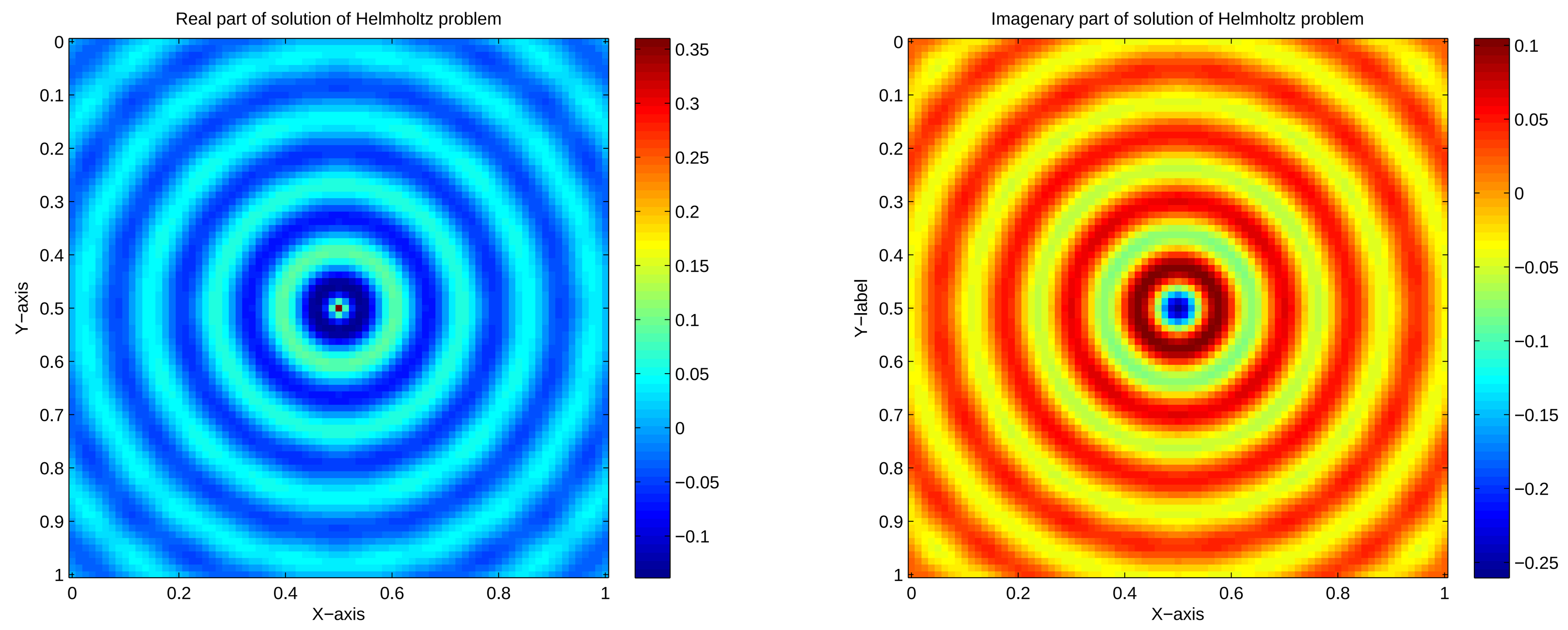

Figure 1: Real *(left)* part and imagenary *(right)* part of solution of the Helmholtz equation solved by GMRES preconditioned with shifted Laplace preconditioner $M(1, 0.1)$.

Helmholtz Model Problem

The Helmholtz equation with Sommerfeld B.cs. is

$$-\Delta \mathbf{u}(x,y) - k^2\mathbf{u}(x,y)\mathbf{u}(x,y) = \mathbf{g}(x,y)$$
$$\left(\frac{\delta u}{\delta n} - \iota k u\right) = 0$$

where $\frac{\delta u}{\delta n}$, the normal derivative of u , $k = \frac{2\pi}{\lambda} = \frac{\omega}{c(x)}$, the wavenumber and $g(x,y)$, the point source function.

Discretizaton leads to 5 diagonal, symmetric, complex valued and indefinite linear system.

Solver

Two-level preconditioned **Krylov subspace solvers** i.e. **GMRES**.
Shifted Laplace preconditioner performs better than available preconditioners for Helmholtz, and comes up near-zero eigenvalues for large wavenumber probelm.Second level preconditioner:
First level preconditioner : Shifted Laplace Preconditioner

$$M_h := -\Delta - (\beta_1 + \iota\beta_2)k^2 I_h \tag{1}$$

Second level preconditioner : Multigrid deflation

$$P_{h,H} = I_h - I_H^h(A_H)^{-1}I_h^H A_h \text{ with } A_H = I_h^H A_h I_H^h . \tag{2}$$

A Good Characteristic

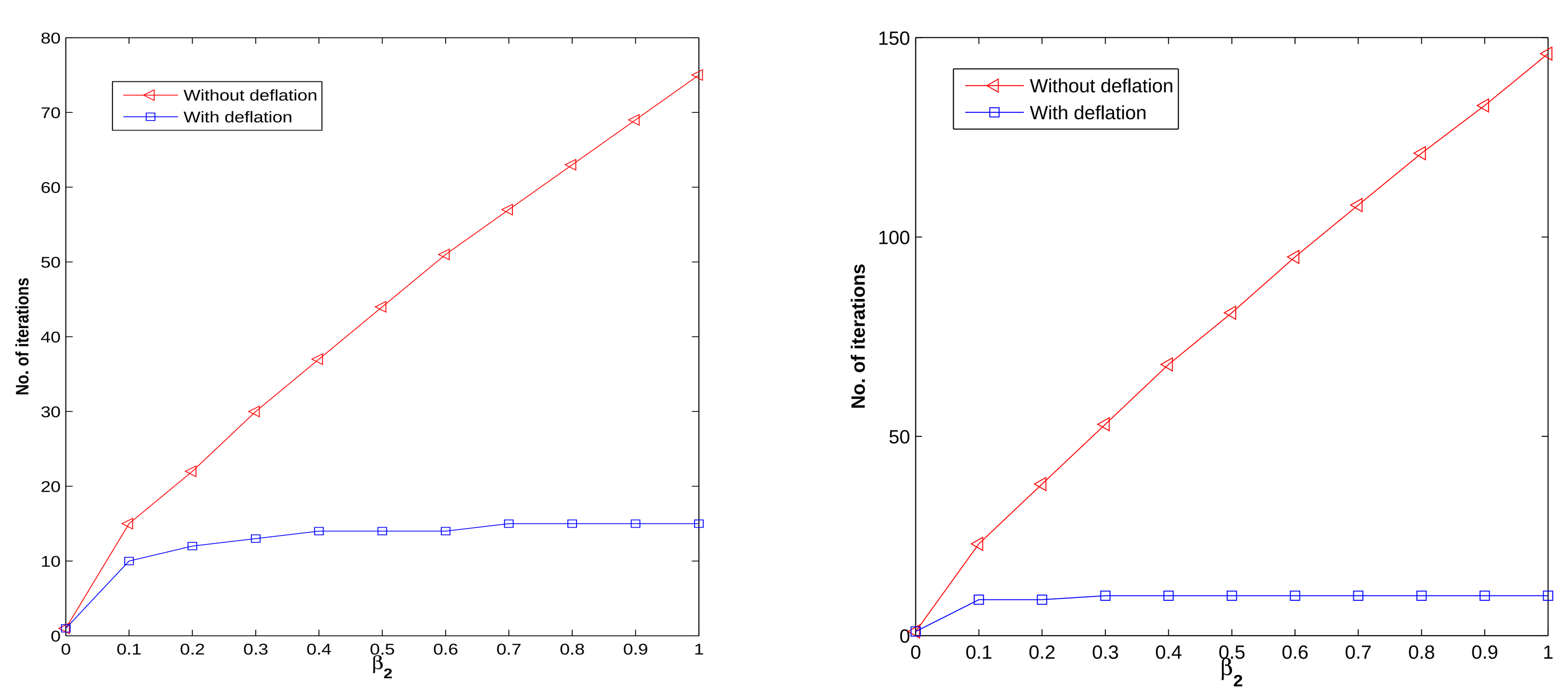

Figure 2: Delfation allows increase in imaginary part of shift in SLP

Multilevel Results

	k = 10	k = 20	k = 40	k = 80	k = 160	320
TL	6	7	11	15	25	50+
MLMGV(4, 2, 1)	9	11	16	27	100+	-
MLMGV(4, 2, 1)*	9	11	15	24	50	-
MLMGV(6, 2, 1)	9	10	14	21	47	-
MLMGV(6, 2, 1)*	9	10	14	20	37	-
MLMGV(8, 2, 1)	9	10	13	20	38	-
MLMGV(8, 2, 1)*	9	10	13	19	29	-
MLMGV(10, 2, 1)	9	10	14	19	32	-

$*$ with damping $\alpha = 0.001$

LFA: 2D Model problem

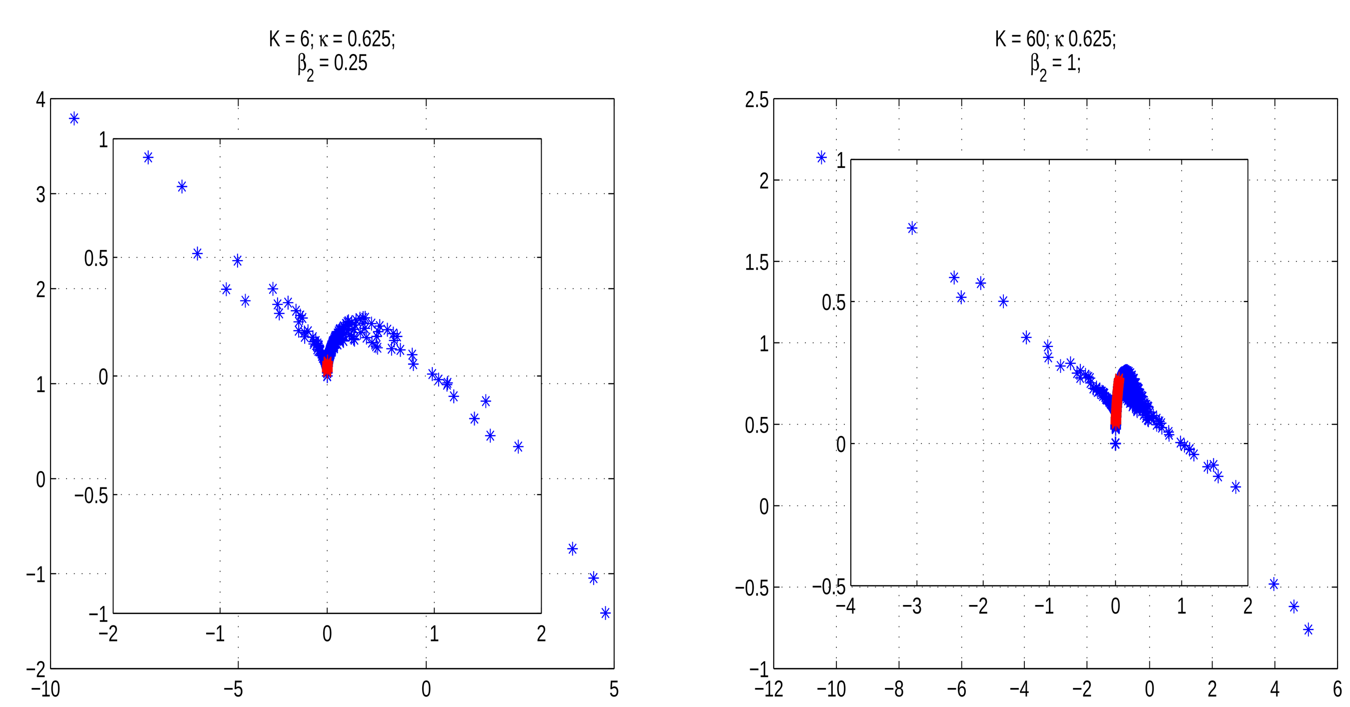

Figure 3: Spectrum of the two grid operator for different values of shift β_2.

Conclusive remarks

- Very slightly dependent.
- More wavenumber is resolved over grid, the more efficient algorithm is.
- Coarse grid solve requires more iteration.
- Increase in imaginary part of shift is privileged by deflation.

References

- Y.A. Erlangga and R. Nabben, ETNA 2008.
- DIAM Tech. Report. 11-01 TU Delft, Netherlands