On convergence of shifted Laplace preconditioner combined with multigrid deflation

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Helmholtz Model Problem

The Helmholtz equation with Sommerfeld B.cs. is

$$ -\Delta u(x, y) - k^2 u(x, y) u(x, y) = g(x, y) $$

$$ \left( \frac{\partial}{\partial n} - i ku \right) = 0 $$

where $\frac{\partial}{\partial n}$, the normal derivative of $u$, $k = \frac{2\pi}{\lambda}$, the wavenumber and $g(x, y)$, the point source function.

Discretization leads to 5 diagonal, symmetric, complex valued and indefinite linear system.

Solver

Two-level preconditioned Krylov subspace solvers i.e. GMRES. Shifted Laplace preconditioner performs better than available preconditioners for Helmholtz, and comes up near-zero eigenvalues for large wavenumber problem. Second level preconditioner: First level preconditioner: Shifted Laplace Preconditioner

$$ M_h := -\Delta - (\beta_1 + i \beta_2) k^2 I_h $$

Second level preconditioner: Multigrid deflation

$$ P_{h,H} = I_h - I_H^H (A_H)^{-1} A_h I_h^H A_h I_H^H $$

A Good Characteristic

<table>
<thead>
<tr>
<th>$k = 10$</th>
<th>$k = 20$</th>
<th>$k = 40$</th>
<th>$k = 80$</th>
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<td>MLMGV(4, 2, 1)</td>
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<td>MLMGV(4, 2, 1)$^*$</td>
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<tr>
<td>MLMGV(10, 2, 1)</td>
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* with damping $\alpha = 0.001$

Conclusive remarks

- Very slightly dependent.
- More wavenumber is resolved over grid, the more efficient algorithm is.
- Coarse grid solve requires more iteration.
- Increase in imaginary part of shift is privileged by deflation.

References

- DIAM Tech. Report. 11-01 TU Delft, Netherlands