Two-Level LFA Multilevel Helmholtz Solver

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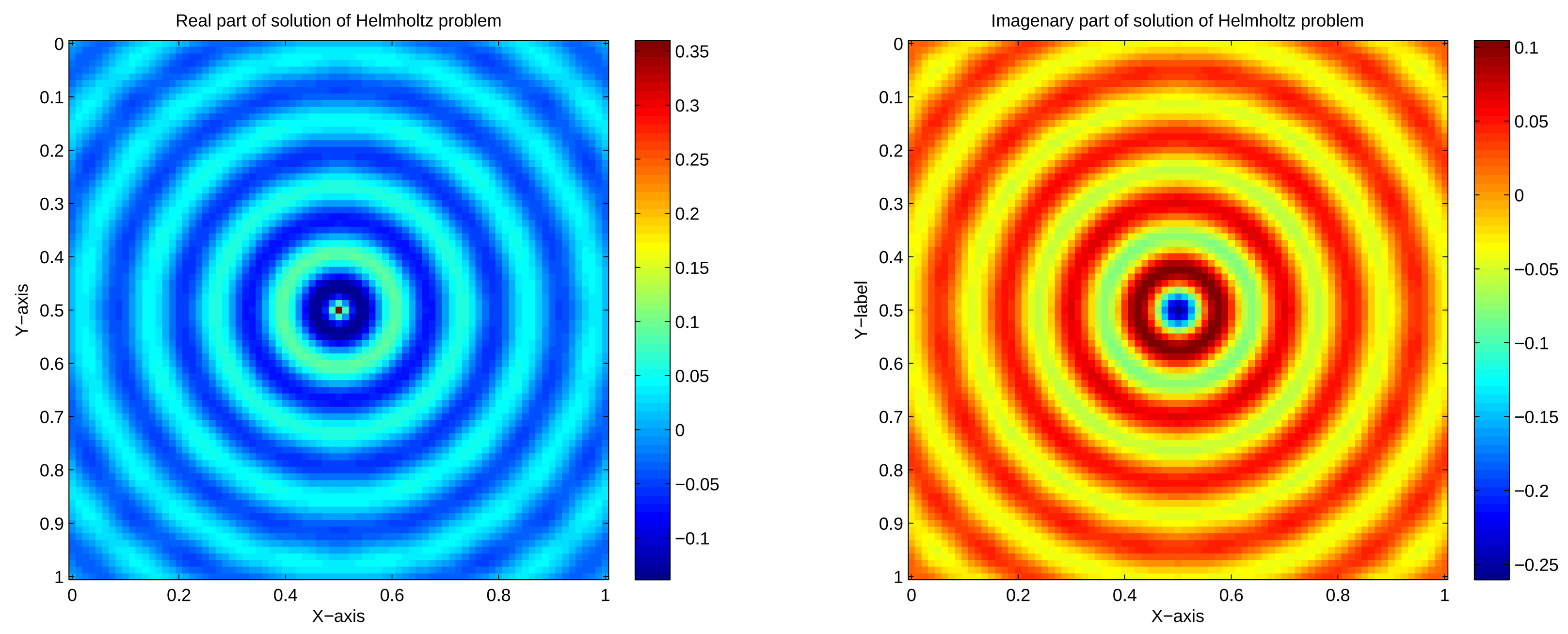

Figure 1: Real *(left)* part and imagenary *(right)* part of solution of the Helmholtz equation solved by GMRES preconditioned with shifted Laplace preconditioner $M(1, 0.1)$.

Helmholtz Equation

The Helmholtz equation on domain Ω reads

$$-\Delta\mathbf{u}(x,y) - k^2(x,y)\mathbf{u}(x,y) = \mathbf{g}(x,y)$$

where $k = \frac{2\pi}{\lambda} = \frac{\omega}{c(x)}$, is the wavenumber.

Discretizaton symmetric, complex valued and highly indefinite linear system for certain large k.

Solver and Preconditioner

Krylov solvers for complex indefinite system (GMRES, GCR, IDR(s), etc), with preconditioners:

First level preconditioner : **Shifted Laplace Preconditioner**

$$M_h := -\Delta - (\beta_1 + \iota\beta_2)k^2 I_h$$

Second level preconditioner : **Multigrid deflation**

$$P_{h,2h} = I_h - Q_H A_h + Q_H$$

where $q_H = I_{2h}^h (A_{2h})^{-1} I_h^{2h}$ and $A_{2h} = I_h^{2h} A_h I_{2h}^h$.

Attraction:

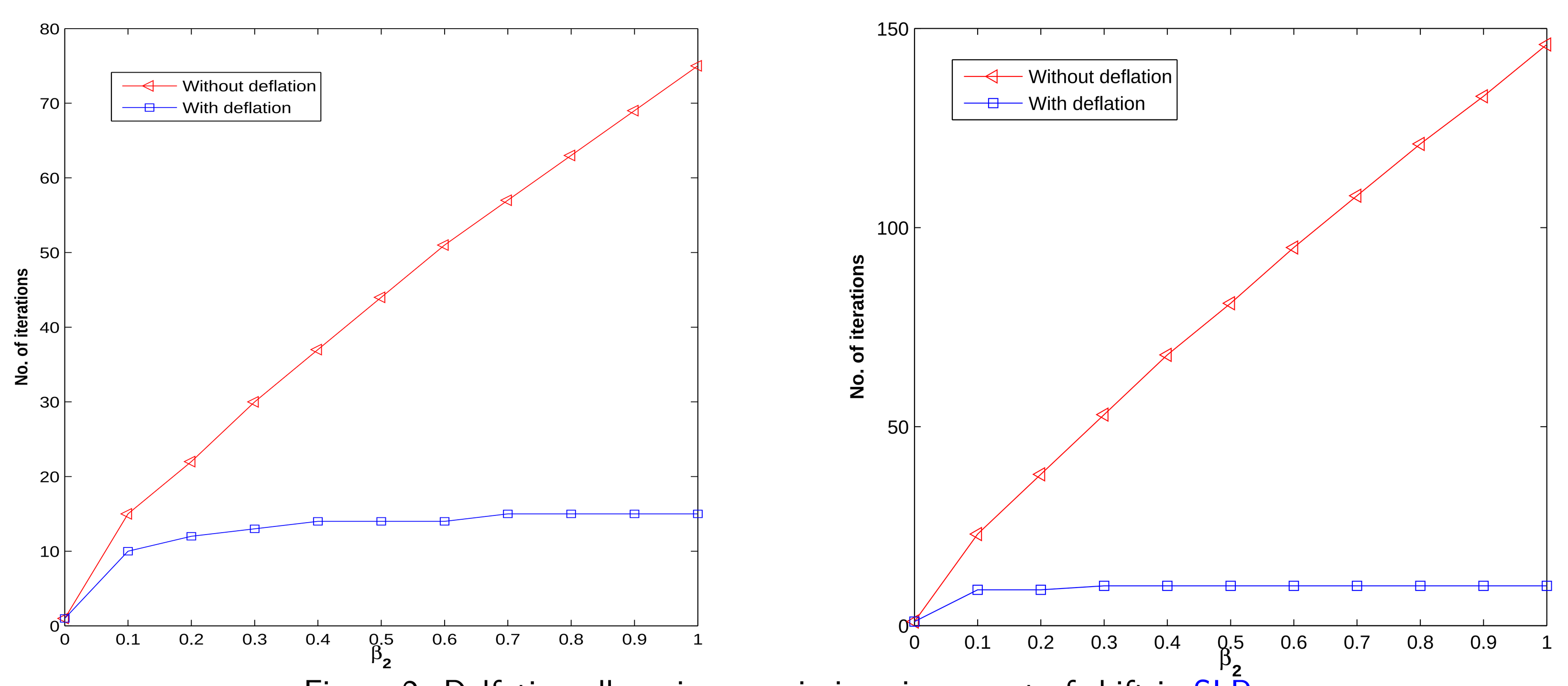

Figure 2: Delfation allows increase in imaginary part of shift in SLP

Multilevel:

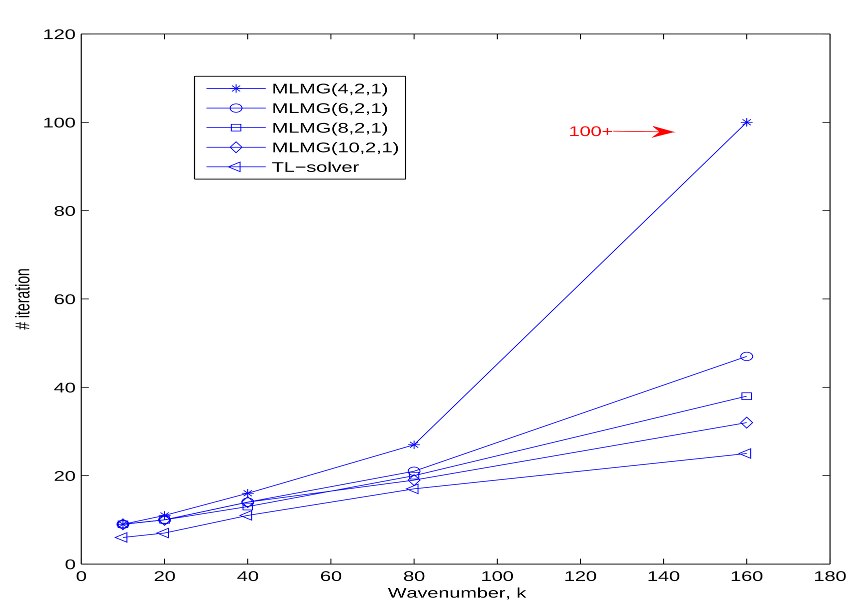

Figure 3: Number of iterations for various solvers for difference values of k.

LFA: 2D Model problem

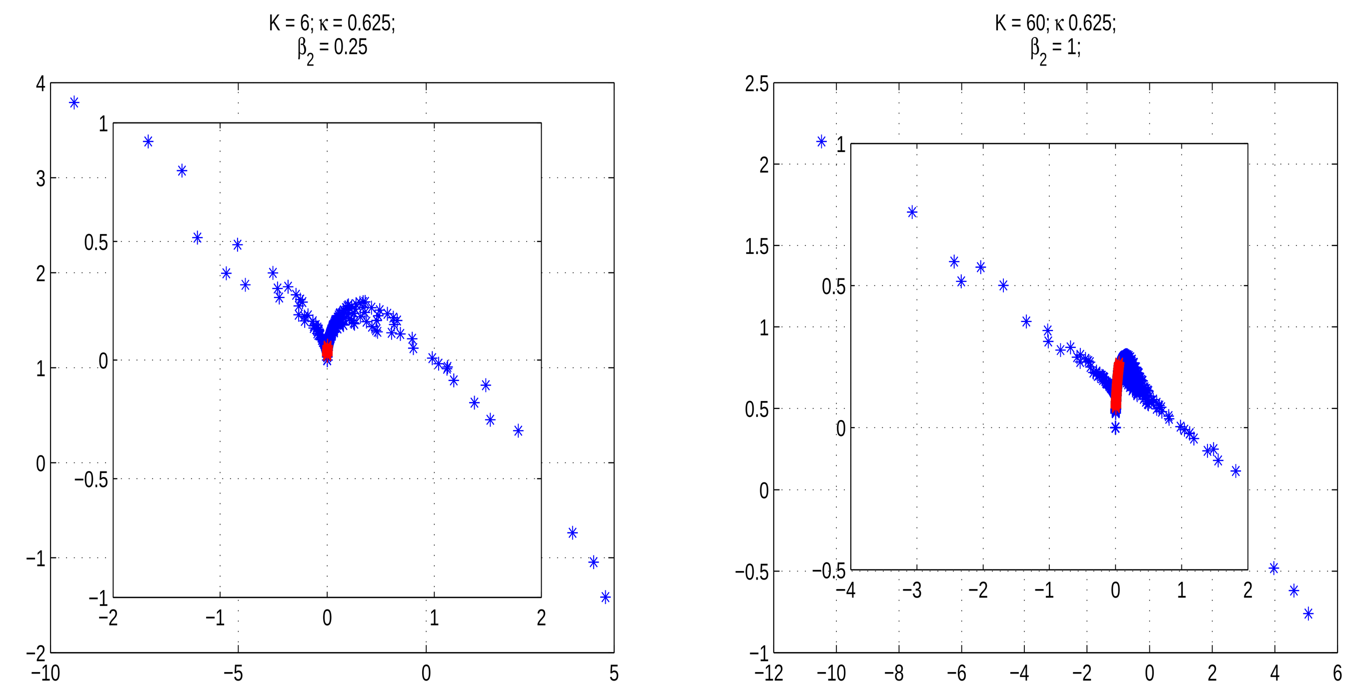

Figure 4: Spectrum of the two grid operator for different values of shift β_2.

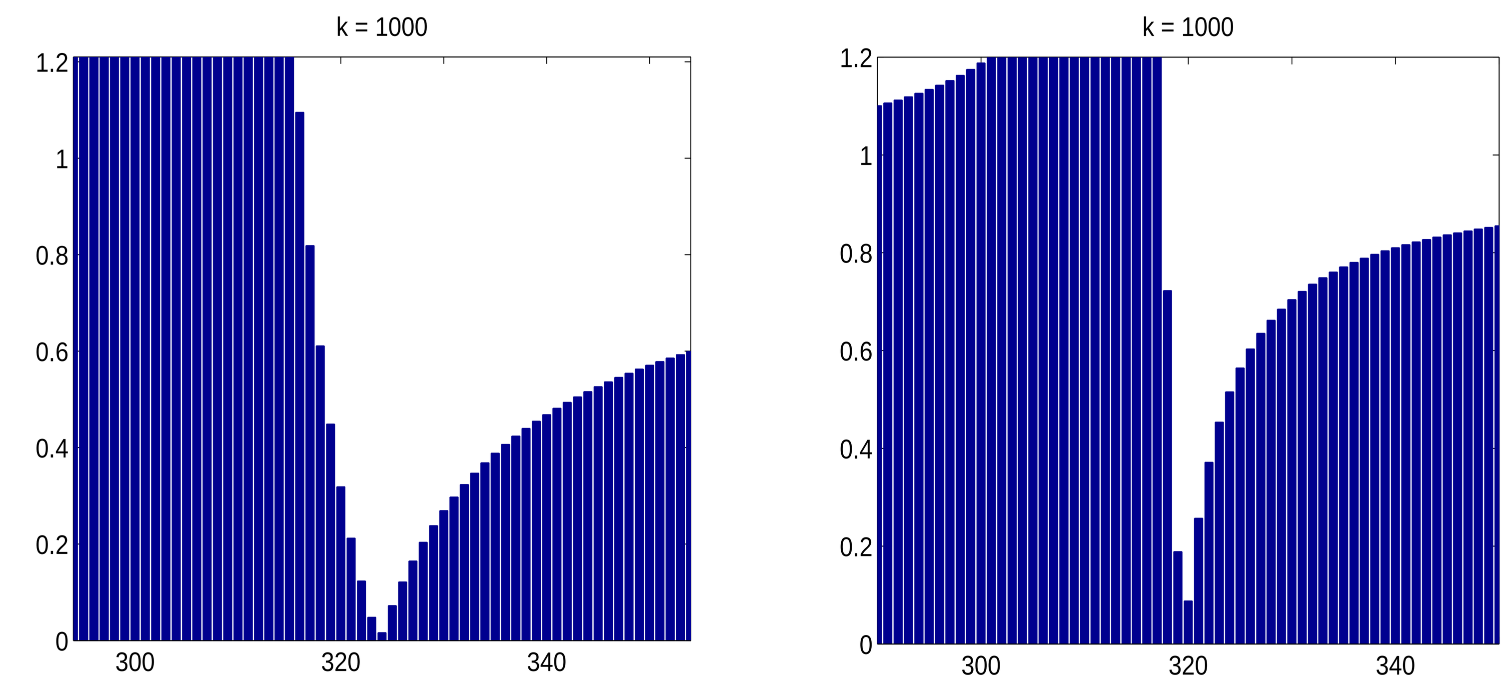

Figure 5: Magnitude of nonzero part of spectrum of the two grid operator for 10 and 20 gp/wl.

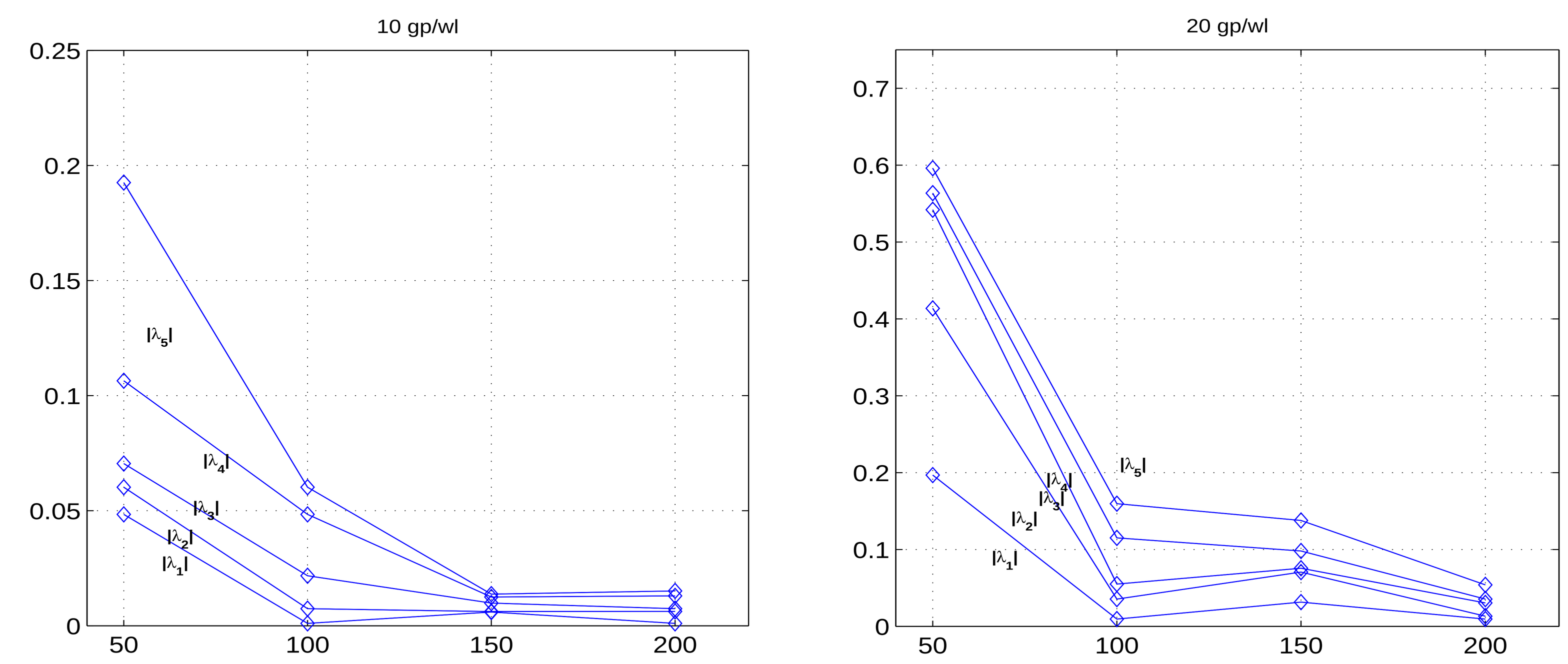

Figure 6: Five smallest eigenvalues of two grid operator for a 2D problem for different k.

Conclusive remarks

- Very slightly dependent.
- More wavenumber is resolved over grid, the more efficient algorithm is.
- Coarse grid solve requires more iteration.
- Increase in imaginary part of shift is privileged by deflation.

References

- Y.A. Erlangga and R. Nabben, ETNA 2008.
- DIAM Tech. Report. 11-01 TU Delft, Netherlands
- A.H.Sheikh, C.Vuik and D. Lahaye, submitted in NLAA.