Simultaneous Linear Equtaions in two and three variables

Linear Algebra - 24 DS

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Today's Menu

- Linear equations in 2 and 3 variables
- Graphical interpretation of solutions
- Simultaneous linear equations
- Geometrical equivalents and solution types

Linear Equation in Two Variables

Definition and General Form

Definition

Linear equations represent straight lines (in 2D). And Solutions represent point(s) of intersection of lines.

General Equation

$$ax + by = c$$

where a, b, and c are real numbers, a and b not both zero.

Example

$$2x + 3y = 6$$

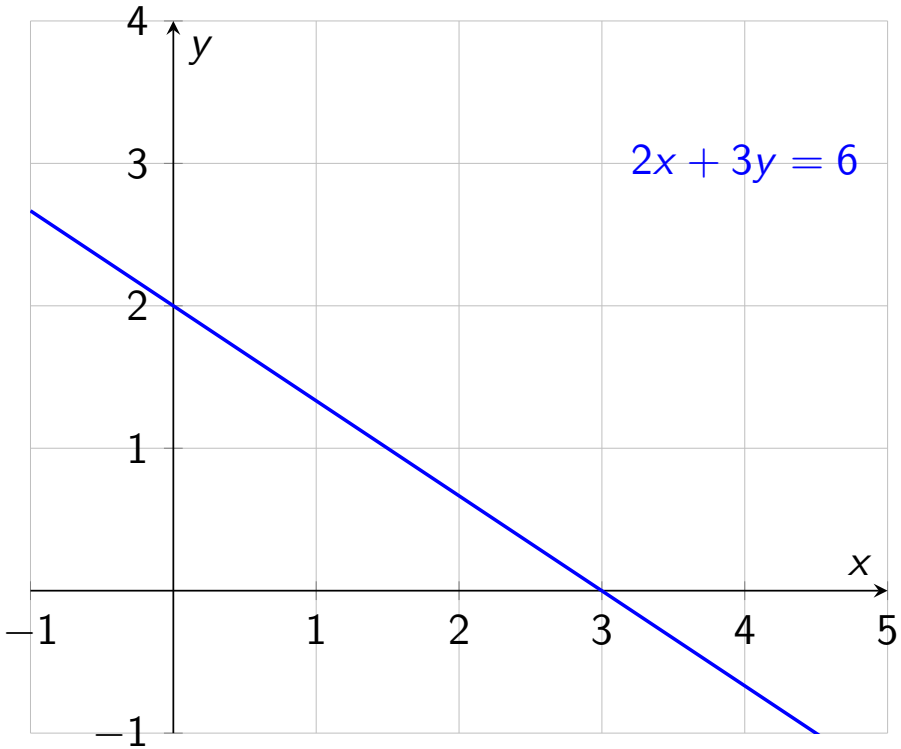
$$x - y = 2$$

Solution

Ordered pair (x, y) that satisfies the equation

Graphical Representation

Single Equation



Every point on the line is a solution to the equation.

Simultaneous Linear Equations in Two Variables

Definition and General Form

Definition

Two or more simultaneous linear equations with two variables is called system of linear equations. .

General Equation

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

where $a_1, b_1, c_1, a_2, b_2,$, and c_2 are real numbers, a_1, b_1, a_2 and b_2 not all zeros.

Example

$$2x + 3y = 6$$
$$x - y = 2$$

Solution

Ordered pair (x, y) that satisfies both equations at the same time.

Linear Equations in Three Variables

General Form

General Equation

$$ax + by + cz = d$$

where a, b, c, and d are real numbers, a, b, c not all zero.

Example

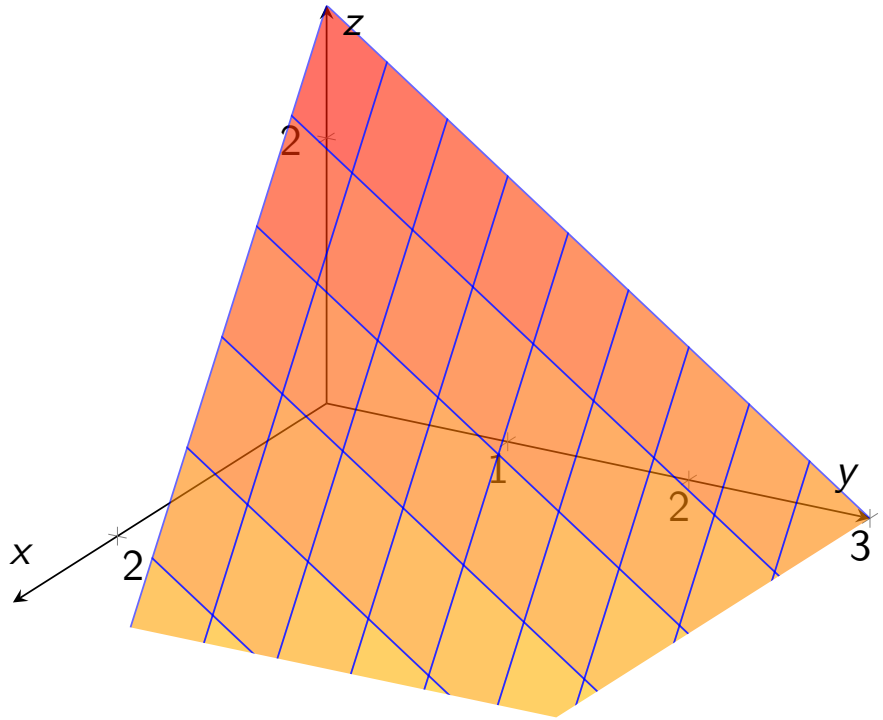
$$2x + y - z = 3$$

$$x - y + 2z = -1$$

$$3x + 2y + z = 4$$

Graphical Interpretation: Planes in 3D

A single linear equation in 3 variables represents a **plane** in 3D space.



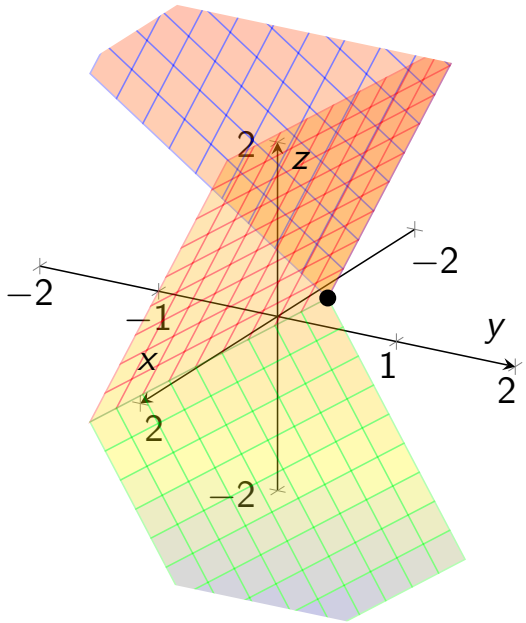
Possible Solutions Scenarios

$$\begin{cases} a_1x + b_1y + c_1z = d_1 \\ a_2x + b_2y + c_2z = d_2 \\ a_3x + b_3y + c_3z = d_3 \end{cases}$$

Possible solution types:

1. **Unique solution** – Three planes intersect at one point
2. **Infinitely many solutions** – Planes intersect along a line or are coincident
3. **No solution** – Planes parallel or intersect pairwise but not all three

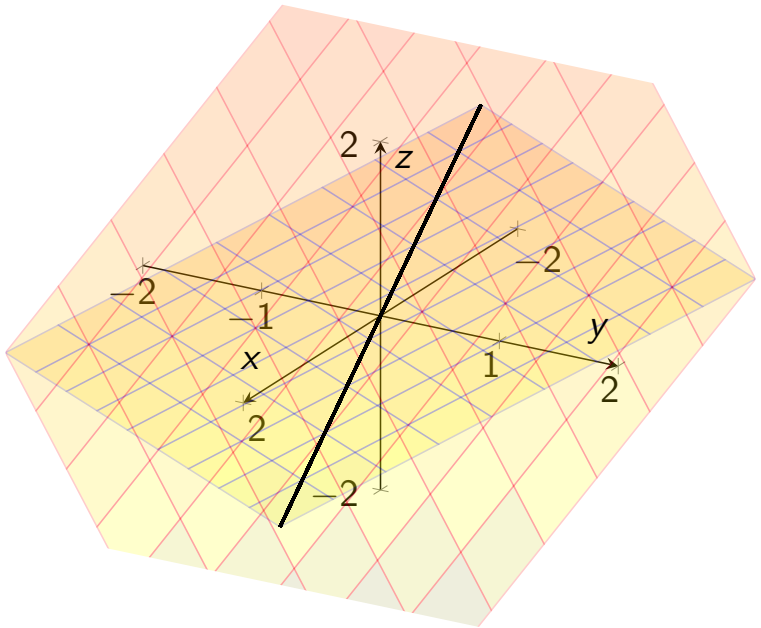
Case 1: Unique Solution (Intersecting Planes)



Three planes intersecting at a single point.

Case 2: Infinitely Many Solutions

Planes Intersect Along a Line

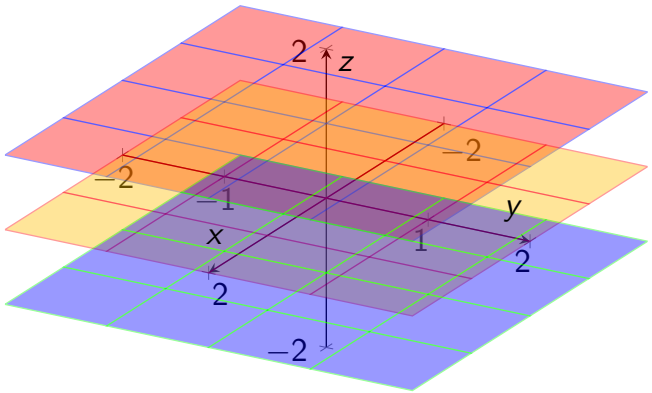


Planes intersect along a common line (infinite solutions).

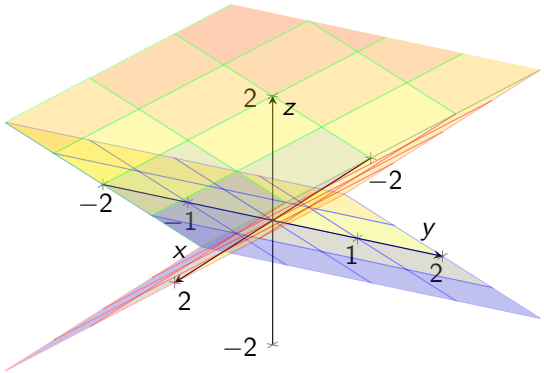
Case 3: No Solution

Parallel or Inconsistent Planes

Parallel Planes:



Pairwise intersections but no common point:



Summary Table

Dimensions	Unique Solution	No Solution	Infinite Solutions
2*2 Variables	Intersecting lines	Parallel lines	Coincident lines
2*3 Variables	Planes intersect at a point	Parallel planes or inconsistent	Planes intersect along a line

Key Concepts

- 2D: Lines intersect at a point, are parallel, or coincide
- 3D: Planes intersect at a point, along a line, are parallel, or coincide
- Solution corresponds to intersection geometry